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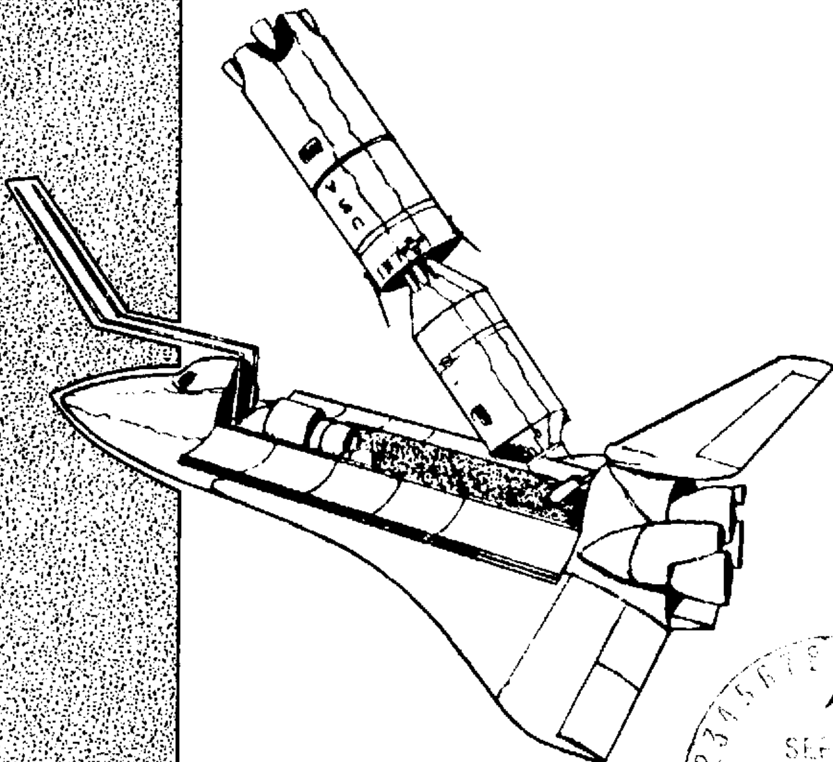
FINAL REPORT
CONTRACT NAS9-13568
JUNE 28, 1974

ASYMMETRICAL BOOSTER ASCENT GUIDANCE AND CONTROL SYSTEM DESIGN STUDY

VOLUME II

SSFS MATH MODELS - ASCENT



REPORT
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BOEING aerospace company
HOUSTON, TEXAS

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ASYMMETRICAL BOOSTER ASCENT GUIDANCE AND CONTROL SYSTEM DESIGN STUDY

VOLUME II
SSFS MATH MODELS - ASCENT

JUNE 28, 1974

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PREFACE

Final report of Asymmetrical Booster Ascent Guidance and Control System Design Studies performed under Contract NAS9-13568 are contained in five separate volumes identified as follows:

- Volume I - Summary
- Volume II - SSFS Math Models - Ascent
- Volume III - Space Shuttle Vehicle SRB Actuator Failure Study
- Volume IV - Sampled Data Stability Analysis Program (SADSAP) Users Guide
- Volume V - Space Shuttle Powered Explicit Guidance

ABSTRACT

This manual presents Boeing developed boost to orbit math models for the NASA/JSC Space Shuttle Functional Simulator.

KEY WORDS

Space Shuttle Vehicle
Math Models
SSFS (Space Shuttle Functional Simulator)
Boost Dynamics
Simulation Models
Flight Dynamics

TABLE OF CONTENTS

1.0 INTRODUCTION

This manual presents the engineering equations and math models developed by the Boeing Aerospace Company for use in the Space Shuttle Functional Simulator (SSFS). These models were originally developed for NASA/JSC under Contract NAS9-12183, and continued under Contract NAS9-13568. This manual contains extensive revisions and additions to earlier documentation and it supersedes the previous math models document, Boeing Memorandum 5-2581-HOU-102 dated 4 October 1972.

Section 2 contains documentation of all Boeing developed math models including several proposed models not yet incorporated into the SSFS. Included in section 3 are definitions of coordinate systems used by the SSFS models and coordinate transformations.

Documentation of the flexible body math models is provided in section 4. These models have been incorporated in the SSFS and are in the checkout stage.

2.0 SSFS MATH MODELS

This section contains environment math models for the SSFS computer program. Several model changes are contained within this documentation which have not yet been incorporated into the program. Subroutine ATTUDE has been deleted as a scheduled model and has been made a part of the flight control system subroutine. The control system now provides the call for attitude commands. Subroutine FLTSEQ (Flight Sequencing Program) has been included to simulate flight control system logic necessary to initiate staging signals.

Each model is discussed under the following format:

X.1 Program Description

X.2 Math Model

X.3 Nomenclature (that used in math model)

X.4 Input/Output Requirements

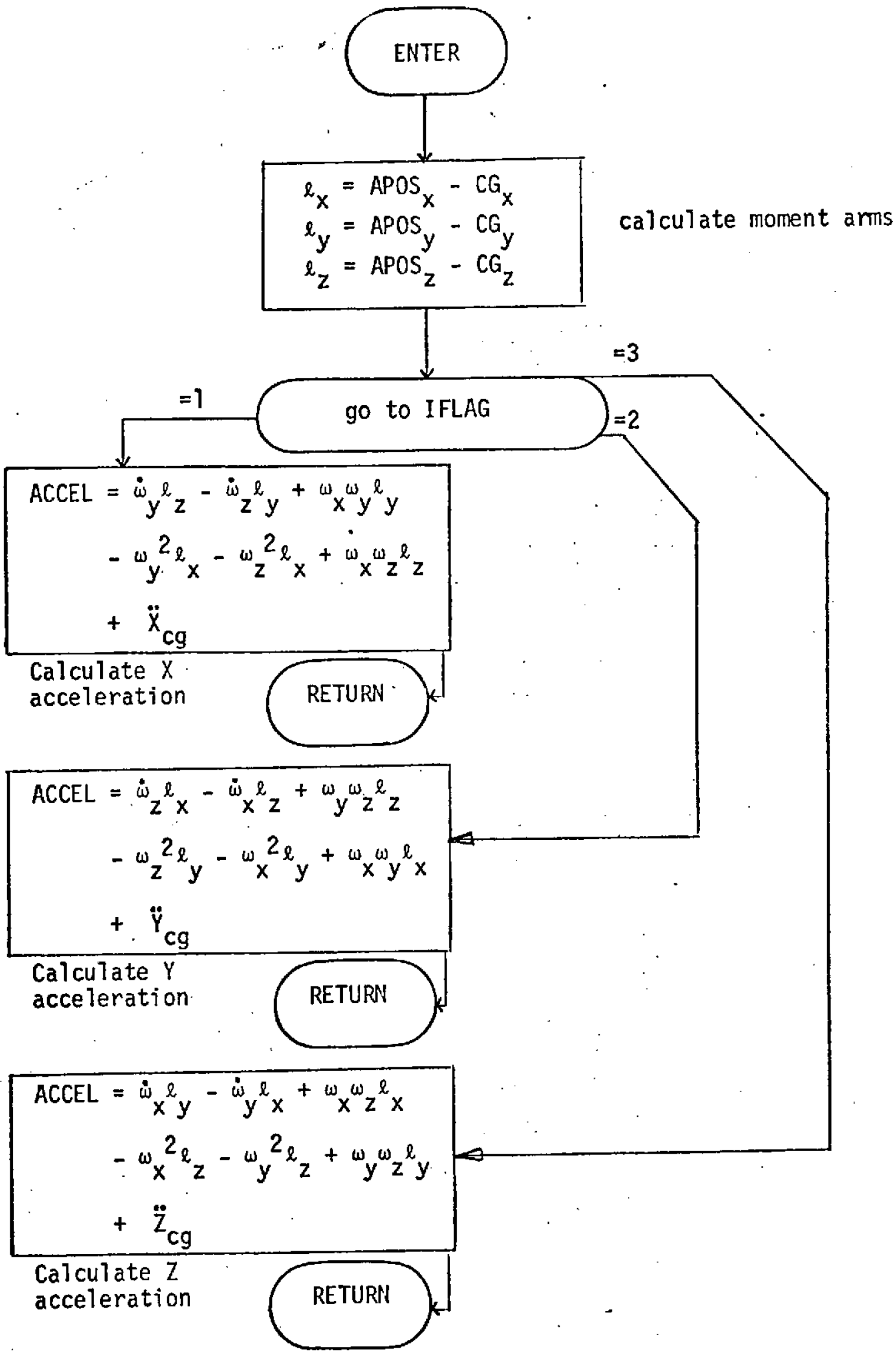
2.1 ACCEL (Acceleration)

2.1.1 Program Description

This routine calculates acceleration in body coordinates for the accelerometer position APOS (X, Y, Z) on the vehicle. The component of acceleration returned is determined by the value of IFLAG. If IFLAG = 1, the X component of acceleration is returned. If IFLAG = 2, the Y component of acceleration is returned. If IFLAG = 3, the Z component of acceleration is returned. All rotational and translational effects are included in the acceleration calculations.

This routine is called by the control routine and is used to generate accelerometer signals for use by the flight control system.

2.1.2 Math Model



2.1.3 Nomenclature

$APOS_x$, $APOS_y$, $APOS_z$	Accelerometer location (X, Y, Z)
CG_x, CG_y, CG_z	Vehicle center of gravity (X, Y, Z)
ℓ_x, ℓ_y, ℓ_z	Moment arms (X, Y, Z)
$\ddot{X}_{cg}$, $\ddot{Y}_{cg}$, $\ddot{Z}_{cg}$	Vehicle center of gravity accel (X, Y, Z)
ω_x, ω_y, ω_z	Vehicle body angular rates (X, Y, Z)
$\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$	Vehicle body angular accelerations (X, Y, Z)
IFLAG	Flag to specify component of acceleration to be returned IFLAG = 1 returns X component IFLAG = 2 returns Y component IFLAG = 3 returns Z component

2.1.4 Input/Output

This routine requires, as formal parameter input, accelerometer location and a code identifying the desired component of acceleration to be returned. Body angular rates, angular accelerations, translational accelerations, and C.G. locations must be input via common. Acceleration of the position denoted is output.

2.2 ACTVEH (6 DOF Equations of Motion)

ACTVEH defines the motions of the center of gravity of the vehicle. For convenience it is separated into five parts; 1) translation equations, 2) rotation equations, 3) euler angles, 4) initial position calculations, and 5) momentum transfer at staging.

These first three equations should be solved at least once each second during powered flight. In the vicinity of environmental discontinuities more frequent solution is required; for instance, the vehicle can fly completely through a wind gust at maximum dynamic pressure within 0.1 second. Other discontinuities include: staging, start of closed loop guidance, and engine or actuator failures. As a rule of thumb, the integration rate during transients can be 1/2 x rotational acceleration (in degrees/sec^2).

2.2.1 Translation Equations

2.2.1.1 Program Description

This model defines the linear accelerations of the rigid body.

2.2.1.2 Math Model

$$\begin{bmatrix} \Sigma F_{X_P} \\ \Sigma F_{Y_P} \\ \Sigma F_{Z_P} \end{bmatrix} = [\beta] \begin{bmatrix} \Sigma F_{X_B} \\ \Sigma F_{Y_B} \\ \Sigma F_{Z_B} \end{bmatrix}$$

$$\begin{bmatrix} g_{X_P} \\ g_{Y_P} \\ g_{Z_P} \end{bmatrix} = [\alpha] \begin{bmatrix} g_{X_I} \\ g_{Y_I} \\ g_{Z_I} \end{bmatrix}$$

$$\ddot{X}_P = g_{X_P} + \Sigma F_{X_P}/m \qquad \dot{X}_P = \int_{t_1}^{t_2} \ddot{X}dt + \dot{X}_{P_o} \qquad X_P = \int_{t_1}^{t_2} \dot{X}dt + X_{P_o}$$

$$\ddot{Y}_P = g_{Y_P} + \Sigma F_{Y_P}/m \qquad \dot{Y}_P = \int_{t_i}^{t_2} \ddot{Y}dt + \dot{Y}_{P_o} \qquad Y_P = \int_{t_1}^{t_2} \dot{X}dt + Y_{P_o}$$

$$\ddot{Z}_P = g_{Z_P} + \Sigma F_{Z_P}/m \qquad \dot{Z}_P = \int_{t_1}^{t_2} \ddot{Z}dt + \dot{Z}_{P_o} \qquad Z_P = \int_{t_1}^{t_2} \dot{Z}dt + Z_{P_o}$$

2.2.1.3 Nomenclature

ΣF_{X_B}, ΣF_{Y_B}, ΣF_{Z_B} = sum of forces in the X, Y, Z body axis directions.

ΣF_{X_P}, ΣF_{Y_P}, ΣF_{Z_P} = sum of forces in the X, Y, Z inertial plumbline axis directions.

$[\beta]$ = transformation matrix from body to inertial plumbline.

ΣF = aero forces + thrust forces + RCS forces + engine deflection forces + slosh forces.

g_{X_I}, g_{Y_I}, g_{Z_I} = gravitational acceleration components in inertial polar-equatorial axis directions.

g_{X_P}, g_{Y_P}, g_{Z_P} = gravitational acceleration components in inertial plumbline axis directions.

$[\alpha]$ = transformation matrix from inertial polar - equatorial to inertial plumbline.

X_P, Y_P, Z_P = accelerations in inertial plumbline axis directions

m = total vehicle mass

2.2.1.4 Input/Output

The translation equations require as inputs:

- Aerodynamic forces
- Thrust forces
- RCS forces
- Engine deflection forces
- Slosh forces
- [α] and [β] matrices
- Gravitational acceleration components
- Vehicle mass
- Initial conditions on $\dot{X}_p$, $\dot{Y}_p$, $\dot{Z}_p$, X_p, Y_p, Z_p

The outputs from the translation equations are:

ΣF_{X_B}, ΣF_{Y_B}, ΣF_{Z_B} and the inertial plumbline position, velocity and acceleration components.

The translation equations require the presence of subroutines: RCS, THRUST, AERO,TVC, SLOSH AND GRAVITY.

2.2.2 Rotational Equations

2.2.2.1 Program Description

This model defines the angular accelerations of the rigid body assuming that the center of mass lies approximately in the X-Z plane ($I_{YZ} \cong I_{XY} \cong 0$).

2.2.2.2 Math Model

$$\dot{q} = \frac{1}{I_{YY}} \quad \Sigma M_{Y_B} + pr\,(I_{ZZ} - I_{XX}) + (r^2 - p^2)\, I_{XZ}$$

$$\dot{p} = (a\, I_{ZZ} + b\, I_{XZ})/C$$

$$\dot{r} = (a\, I_{XZ} + b\, I_{XX})/C$$

$$a = \Sigma M_{X_B} + qr\,(I_{YY} - I_{ZZ}) + pq\, I_{XZ}$$

$$b = \Sigma M_{Z_B} + pq\,(I_{XX} - I_{YY}) - qr\, I_{XZ}$$

$$c = I_{XX}\, I_{ZZ} - I_{XZ}^2$$

2.2.2.3 Nomenclature

$\dot{q}$ = Angular acceleration about the Y body axis

$\dot{p}$ = Angular acceleration about the X body axis

$\dot{r}$ = Angular acceleration about the Z body axis

I_{XX}, I_{YY}, I_{ZZ} = Moment of inertia about X, Y, Z body axis respectively.

I_{XZ} = X - Z Cross product moments of inertia

p, q, r = Integral of $\dot{p}, \dot{q}, \dot{r}$ (Body rates)

$\Sigma M_{X_B}, \Sigma M_{Y_B}, \Sigma M_{Z_B}$ = Sum of moments about X, Y, Z body axes

= Aero moments + thrust moments + RCS moments + engine deflection moments + slosh moments

2.2.2.4 Input/Output

Inputs: From THRUST, AERO, RCS, TVC and SLOSH

Moments (about body axes) due to aerodynamics, main propulsion, reaction control, engine accelerations and slosh.

Outputs: $\dot{P}, \dot{Q}, \dot{R}$, P Q, R to IMU, Aero and Euler Angles

2.2.3 Euler Angles

2.2.3.1 Program Description

This model defines the rate of change of the euler angles describing the attitude of the vehicle in inertial space.

2.2.3.2 Math Model

$\dot{\theta} = (q \cos \phi - r \sin \phi)/\cos \psi$

$\dot{\psi} = q \sin \phi + r \cos \phi$

$\dot{\phi} = p = \tan \psi (q \cos \phi + r \sin\phi)$

2.2.3.3 Nomenclature

$\dot{\theta}$, $\dot{\psi}$, $\dot{\phi}$ = Euler angle rates (1st, 2nd, and 3rd rotations, respectively)

θ, ψ, ϕ = Integral of $\dot{\theta}$, $\dot{\psi}$, $\dot{\phi}$

2.2.3.4 Input/Output

Inputs p, q, r from rotation equations

Outputs θ, ψ, ϕ to [β] and to IMU

2.2.4 Initial Position Math Model

2.2.4.1 Program Description

This program calculates the difference between geodetic and geocentric latitude and uses it to calculate the initial state vector. This calculation needs to be done once each time either the launch azimuth or latitude is changed.

2.2.4.2 Math Model

$$\psi_L = \text{Arctan}\left[(1-f)^2 \tan \phi_L\right]$$

$$R_L = \frac{R_e\ (1-f)}{\sqrt{1-f\ (2-f)\ \cos^2 \psi_L}} + ho$$

$$\beta = \phi_L - \psi_L$$

$$R_X = R_L \cos \beta$$

$$R_Y = R_L \sin \beta \sin A_Z$$

$$R_Z = -R_L \sin \beta \cos A_Z$$

$$V_X = 0$$

$$V_Y = \omega\, R_L \cos A_Z \cos \psi_L$$

$$V_Z = \omega\, R_L \sin A_Z \cos \psi_L$$

2.2.4.3 Nomenclature

ω = rotation rate of earth

ϕ_L = launch geodetic latitude

f = earth flattening constant

R_e = earth equatorial radius

A_Z = launch azimuth

ψ_L = launch geocentric latitude

R_L = magnitude of initial position vector

β = difference between geodetic & geocentric latitude

$$\left.\begin{matrix} R_X \\ R_Y \\ R_Z \end{matrix}\right\} = \text{initial position vector in platform coordinate}$$

$$\left.\begin{matrix} V_X \\ V_Y \\ V_Z \end{matrix}\right\} = \text{initial velocity vector in platform coordinate}$$

ho = altitude of vehicle CG above Fischer ellipse

2.2.4.4 Input/Output

The constants needed by this model are ω, ϕ_L, f, R_e, and A_Z. The output of the program is R_X, R_Y, R_Z, V_X, V_Y, and V_Z.

2.2.5 Staging Momentum Transfer

2.2.5.1 Program Description

This model accomplishes momentum transfer to the orbiter at the time of staging.

2.2.5.2 Math Model

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} X_{LC} \\ Y_{LC} \\ Z_{LC} \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} \Delta CG_Y \\ \Delta CG_Y \\ \Delta CG_Z \end{bmatrix}$$

$$\begin{bmatrix} \dot{X}_0 \\ \dot{Y}_0 \\ \dot{Z}_0 \end{bmatrix} = \begin{bmatrix} \dot{X}_{LC} \\ \dot{Y}_{LC} \\ \dot{Z}_{LC} \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} r_{LC}\Delta CG_y - q_{LC}\Delta CG_z \\ P_{LC}\Delta CG_z - r_{LC}\Delta CG_x \\ q_{LC}\Delta CG_x - P_{LC}\Delta CG_y \end{bmatrix}$$

2.2.5.3 Nomenclature

X_0, Y_0, Z_0	Orbiter inertial positions immediately after separation
X_{LC}, Y_{LC}, Z_{LC}	Launch configuration inertial position immediately before separation
$\dot{X}_0, \dot{Y}_0, \dot{Z}_0$	Orbiter inertial linear velocity components immediately after separation
$\dot{X}_{LC}, \dot{Y}_{LC}, \dot{Z}_{LC}$	Launch configuration linear velocity immediately before separation
$[B]$	Body to inertial transformation matrix
P_{LC}	Launch configuration angular rotation about X body axis immediately before separation
q_{LS}	Launch configuration angular rotation about Y body axis immediately before separation
r_{LS}	Launch configuration angular rotation about Z body axis immediately before separation

$X_{CG_{LC}}$, $Y_{CG_{LC}}$, $Z_{CG_{LC}}$	Center-of-gravity location in body coordinates for launch configuration immediately before separation
X_{CG_0}, Y_{CG_0}, Z_{CG_0}	Center-of-gravity location in body coordinates for orbiter immediately after separation
ΔCG_X	$\lvert X_{CG_{LC}} - X_{CG_0} \rvert$ pre-separation
ΔCG_Y	$\lvert Y_{CG_{LC}} - Y_{CG_0} \rvert$ pre-separation
ΔCG_Z	$\lvert Z_{CG_{LC}} - Z_{CG_0} \rvert$ pre-separation

2.2.5.4 Input/Output

Inputs: Vehicle Pre-separation C.G. Locations

Vehicle Pre-separation Inertial Position

Vehicle Pre-separation Velocity Components

Vehicle Pre-separation Angular Rates

from MASPRO and ACTVEH (translational and rotational sections)

Outputs: Orbiter inertial position and velocity components

2.3 AERO (Aerodynamics)

2.3.1 Program Description

This model calculates and sums aerodynamic forces and moments for the vehicle. In addition this model calculates the latitude and longitude of the vehicle, flight path angle, mach number, dynamic pressure, angle-of-attack and sideslip angle, and the contribution to velocity due to wind speed and direction.

2.3.2 Math Model

$$\begin{bmatrix} X_F \\ Y_F \\ Z_F \end{bmatrix} = \left[A\right]^T \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix}$$

$$\lambda_V = \sin^{-1} (Z_F/R_V)$$

$$\phi = \tan^{-1}\left(\frac{Y_F}{X_F}\right) - \omega_e (t_L+t)$$

$$V_{EARTH_X} = -W_e Y_F$$

$$V_{EARTH_Y} = W_e X_F$$

$$\begin{bmatrix} V_{R_{X_P}} \\ V_{R_{Y_P}} \\ V_{R_{Z_P}} \end{bmatrix} = \begin{bmatrix} \dot{X}_P \\ \dot{Y}_P \\ \dot{Z}_P \end{bmatrix} - \left[A\right] \begin{bmatrix} V_{EARTH_X} \\ V_{EARTH_Y} \\ 0 \end{bmatrix}$$

$$V_{R_P} = \left(V_{R_{X_P}}^{\ 2} + V_{R_{Y_P}}^{\ 2} + V_{R_{Z_P}}^{\ 2}\right)^{\frac{1}{2}}$$

$$\begin{bmatrix} V_{X_{LV}} \\ V_{Y_{LV}} \\ V_{Z_{LV}} \end{bmatrix} = \begin{bmatrix} D \end{bmatrix}^T \begin{bmatrix} A \end{bmatrix}^T \begin{bmatrix} V_{R_{XP}} \\ V_{R_{YP}} \\ V_{R_{ZP}} \end{bmatrix}$$

$$\gamma = \sin^{-1} \left(V_{X_{LV}} / V_{R_P}\right)$$

$$V_W = \text{table lookup} \sim f(\text{altitude})$$

$$AZ_W = \text{table lookup} \sim f(\text{altitude})$$

$$\begin{bmatrix} V_{W_{XP}} \\ V_{W_{YP}} \\ V_{W_{ZP}} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} 0 \\ -V_W \sin AZ_W \\ -V_W \cos AZ_W \end{bmatrix}$$

$$\begin{bmatrix} V_{R_{X_B}} \\ V_{R_{Y_B}} \\ V_{R_{X_B}} \end{bmatrix} = \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} V_{R_{X_P}} - V_{W_{X_P}} \\ V_{R_{Y_P}} - V_{W_{Y_P}} \\ V_{R_{Z_P}} - V_{W_{Z_P}} \end{bmatrix}$$

$$V_B = \left(V_{R_{X_B}}{}^2 + V_{R_{Y_B}}{}^2 + V_{R_{Z_B}}{}^2 \right)^{\frac{1}{2}}$$

$$\beta = \sin^{-1} \left(V_{R_{Y_B}} / V_B \right)$$

$$\alpha = \tan^{-1} \left(V_{R_{Z_B}} / V_{R_{X_B}} \right)$$

Obtain from ATMØS

1) $\rho = f$ (altitude)
2) $a = f$ (altitude)
3) $P = f$ (altitude)

$M = V_B / a$

$q = 1/2 \, \rho \, V_B^2$

The following aerodynamic coefficients are looked up in tables for lift-off, boost to SRB separation, or orbiter + ET to MECO.

$C_{Z_0} = f\ (M)$

$C_{Z_\alpha} = f\ (M, \alpha)$

$C_{M_0} = f\ (M)$

$C_{M_\alpha} = f\ (M, \alpha)$

$C_{X_0} = f\ (M)$

$C_{X_\alpha} = f\ (M)$

$C_{Y_\beta} = f\ (M)$

$C_{l_\beta} = f\ (M)$

$C_{n_\beta} = f\ (M)$

$C_{n_p} = f\ (M)$

$C_{l_p} = f\ (M)$

$C_{l_r} = f\ (M)$

$C_{n_{\delta_a}} = f\ (M)$

$C_{l_{\delta_r}} = f\ (M)$

$C_{n_{\delta_r}} = f\ (M)$

$C_{l_{\delta_a}} = f\ (M)$

Criteria for the selection of which group of tables to use should be as follows:

1) If $M < 0.6$, use liftoff aero data tables.
2) If $M > 0.6$ and prior to SRB separation, use orbiter + ET + SRB aero data tables.
3) If time is past SRB separation time, use orbiter + ET aero data tables.

Compute aerodynamic forces and moments

$$F_{A_X} = qS\,(C_{X_0} + C_{X_\alpha}\,\alpha)$$

$$F_{A_Y} = qS\,C_{Y_\beta}\,\beta + \frac{q\,S\,b}{2\,V_{R_{X_B}}}\,C_{Y_P}\,P + q\,S\,C_{Y_{\delta r}}\,\delta r$$

$$F_{A_Z} = qS\,(C_{Z_0} + C_{Z_\alpha}\,\alpha) + q\,S\,C_{Z_{\delta e}}\,\delta e$$

$$M_{A_X} = F_{A_Y}\,(Z_{CG} - Z_{A_R}) - F_{A_Z}\,(Y_{CG} - Y_{A_R}) + q\,S\,b\,(C_{l_\beta}\,\beta +$$

$$C_{l_{\delta a}}\,\delta a + C_{l_{\delta r}}\,\delta r) + \frac{q\,S\,b^2}{2V_{R_{X_B}}}\,(C_{l_P}\,P + C_{l_r}\,R)$$

$$M_{A_Y} = q\,S\,\bar{c}\,(C_{m_0} + C_{m_\alpha}\,\alpha) + F_{A_Z}\,(X_{CG} - X_{A_R}) - F_{A_X}\,(Z_{CG} - Z_{A_R})$$

$$+ q\,S\,\bar{c}\left(C_{m_{\delta e}}\,\delta e + C_{m_q}\,Qa/2V_{R_{X_B}}\right)$$

$$M_{A_Z} = q\,S\,b\,C_{n_\beta}\,\beta - F_{A_Y}\,(X_{CG} - X_{A_R}) + F_{A_X}\,(Y_{CG} - Y_{A_R})$$

$$+ \frac{q\,S\,b^2}{2\,V_{R_{X_B}}}\,C_{n_P}\,P - q\,S\,b\,(C_{n_{\delta a}}\,\delta a + C_{n_{\delta r}}\,\delta r)$$

2.3.3 Nomenclature

X_F, Y_F, Z_F = vehicle position in inertial polar-equatorial coordinates

X_P, Y_P, Z_P = vehicle position in inertial plumbline coordinates

$[A]$ = transformation matrix from inertial polar-equatorial to plumbline coordinates

λ_V = latitude of present position of vehicle

ϕ = East longitude of present position of vehicle corrected for earth's rotation

t_L = time of launch (from epoch)

C_1 = radians to degrees conversion constant

W_e = angular rate of earth

t = elapsed time from liftoff

R_V = distance from the center of the earth to the vehicle

V_{EARTH_X}, V_{EARTH_Y} = components of earth's velocity in inertial polar-equatorial coordinates

$V_{R_{X_P}}$, $V_{R_{Y_P}}$, $V_{R_{Z_P}}$ = components of vehicle relative velocity in plumbline coordinates

$\dot{X}_P$, $\dot{Y}_P$, $\dot{Z}_P$ = components of vehicle velocity in plumbline coordinates

V_{R_P} = total vehicle relative velocity in plumbline coordinates

$V_{X_{LV}}$, $V_{Y_{LV}}$, $V_{Z_{LV}}$ = components of relative velocity in local vertical coordinates

$[D]$ = transformation matrix from local vertical to inertial polar-equatorial coordinates

γ = vehicle flight path angle with respect to local horizontal

V_W = horizontal wind speed in local vertical coordinates

AZ_W = wind azimuth (North = 0^o)

$V_{W_{X_P}}$, $V_{W_{Y_P}}$, $V_{W_{Z_P}}$ = components of wind velocity in plumbline coordinates

$V_{R_{X_B}}$, $V_{R_{Y_B}}$, $V_{R_{Z_B}}$ = vehicle velocity with respect to air in body coordinates

$[B]$ = transformation matrix from body to plumbline coordinates

V_B = total vehicle velocity with respect to air in body coordinates

α	=	vehicle angle of attack
β	=	vehicle sideslip angle
ρ	=	local air mass density
a	=	local speed of sound
p	=	local air pressure
M	=	Mach number
q	=	dynamic pressure
F_{A_X}, F_{A_Y}, F_{A_Z}	=	components of aerodynamic force in body coordinates
S	=	vehicle aerodynamic reference area
$\bar{c}$	=	vehicle mean aerodynamic chord
b	=	vehicle reference span
δ_a	=	aileron deflection
δ_e	=	elevator deflection
δ_r	=	rudder deflection
P	=	vehicle roll rate
Q	=	vehicle pitch rate
R	=	vehicle yaw rate
M_{A_X}, M_{A_Y}, M_{A_Z}	=	Aerodynamic moments about the X, Y, and Z body axes, respectively

2.3.4 Input/Output

Input from routines:

X_P, Y_P, Z_P	vehicle position in inertial plumbline coordinates from EOM
$\dot{X}_P$, $\dot{Y}_P$, $\dot{Z}_P$	vehicle velocity in inertial plumbline coordinates from EOM
V_W	wind velocity from tables
AZ_W	wind azimuth from tables
ρ, a, p	current air density, speed of sound and air pressure from ATMOS
X_{CG}, Y_{CG}, Z_{CG}	current location of vehicle center of gravity from MASPRO
t	elapsed time from liftoff from flight sequencer
R_V	distance from center of the earth to the vehicle from EØM
δ_a, δ_e, δ_R	aerodynamic control surface deflections from flight software commands
P, Q, R	vehicle roll, pitch and yaw rates from EØM

All aerodynamic coefficients are input from tables.

Input from cards for initialization:

t_L	time of launch (from epoch)
C_1	radians to degrees conversion constant
W_e	angular rate of earth
S	vehicle aerodynamic reference area
$\bar{c}$	vehicle mean aerodynamic chord
b	vehicle reference span
X_{A_R}, Y_{A_R}, Z_{A_R}	aerodynamic reference location in body coordinates

Output to routines:

p	current air pressure to THRUST
F_{A_X}, F_{A_Y}, F_{A_Z}	components of aerodynamic forces to the EØM
M_{A_X}, M_{A_Y}, M_{A_Z}	moments due to aerodynamic forces to EØM

Output to printer:

λ_V	latitude of vehicle's position
ϕ	longitude of vehicle's position
$V_{X_{LV}}$	rate of climb
γ	flight path angle
V_W	wind speed
AZ_W	wind azimuth
V_B	vehicle velocity with respect to air
α	angle of attack
β	angle of sideslip
ρ	local air mass density
a	local speed of sound
p	local air pressure
M	Mach number
q	dynamic pressure
δ_e, δ_r, δ_a	aerodynamic control surface deflections

2.4 ATMOS (Atmosphere)

2.4.1 Program Description

This program calculates the speed of sound, pressure and air density from an altitude input.

2.4.2 Math Model

Use the Cape Kennedy Reference Atmosphere (TM-X-53872, PARAGRAPH 14.7 - MSFC "COMPUTER SUBROUTINE PRA-63") as specified for SSV design studies.

2.4.3 Input/Output

The altitude above the mean earth surface must be supplied to the model which returns the speed of sound, pressure, and atmospheric density.

2.5 BLC (Baseline Control System)

2.5.1 Program Description

This model issues commands to the engine gimbals (to subroutine TVC via CMDFIL) such that the actual vehicle attitude is made to follow the attitude prescribed by the guidance model. This documentation represents the implementation of the RI system as described in the July 73 Space Shuttle Guidance and Control Data Book, with the addition of roll rate crossfeed into the Y accelerometer channel. With proper input of gain tables, this routine may be used both for first and second stage control.

2.5.2 Math Model

ENTER

IF $(t_{R_2} > t > t_{R_1})$

then

$$\left.\begin{array}{l} AOP\ (1\text{-}8) = \\ AOY\ (1\text{-}8) = \\ RP\ \ (1\text{-}7) = \\ RR\ \ (1\text{-}2) = \\ RY\ \ (1\text{-}7) = \end{array}\right\}\ \text{RAMP TO UPPER STAGE VALUES}$$

PRE-SEPARATION RAMP MIXING LOGIC COEFS. TO UPPER STAGE VALUES

Θ_{CI} = Interpolated from tables UNTIL GUIDANCE START - AND FROM GUIDE THEREAFTER

Ψ_{CI} = Interpolated from tables UNTIL GUIDANCE START - AND FROM GUIDE THEREAFTER

ϕ_{CI} = Interpolated from tables UNTIL GUIDANCE START - AND FROM GUIDE THEREAFTER

$\dot{\gamma}_{CI}$ = Interpolated from tables

DETERMINE COMMAND SIGNALS

$$\phi_{CI} = e^{-K\Delta t} * \phi_{CI} + (1-e^{-K\Delta t}) + \phi_{CI} \text{ (test pass)}$$

FILTER ROLL COMMAND

A

A

KAQ = KAQ (t)
KZ = KZ (t)
KT = KT (t)
KØ = KØ (t)
KY = KY (t)
KAR = KAR (t)
$TRIM_o = TRIM_o \; (V)$
$TRIM_s = TRIM_s \; (V)$

INTERPOLATE CONTROL GAINS AND ENGINE TRIM COMMANDS AS A FUNCTION OF TIME OR VEHICLE RELATIVE VELOCITY

$$\phi_{\varepsilon I} = \phi_I - \phi_{CI}$$
$$\Theta_{\varepsilon I} = \Theta_I - \Theta_{CI}$$
$$\psi_{\varepsilon I} = \psi_I - \psi_{CI}$$

CALCULATE INERTIAL ATTITUDE ERRORS

$$\phi_{\varepsilon B} = \phi_{\varepsilon I} + \Theta_{\varepsilon I} \; SIN \; \psi_I$$
$$\Theta_{\varepsilon B} = \Theta_{\varepsilon I} \; COS \; \phi_I \; COS \; \psi_I + \psi_{\varepsilon I} \; SIN \; \psi_I$$
$$\psi_{\varepsilon B} = \psi_{\varepsilon I} \; COS \; \psi_I - \Theta_{\varepsilon I} \; SIN \; \psi_I \; COS \; \psi_I$$

TRANSFORM TO BODY ERRORS

CALL FLTSEQ

CALL FLIGHT SEQUENCING LOGIC

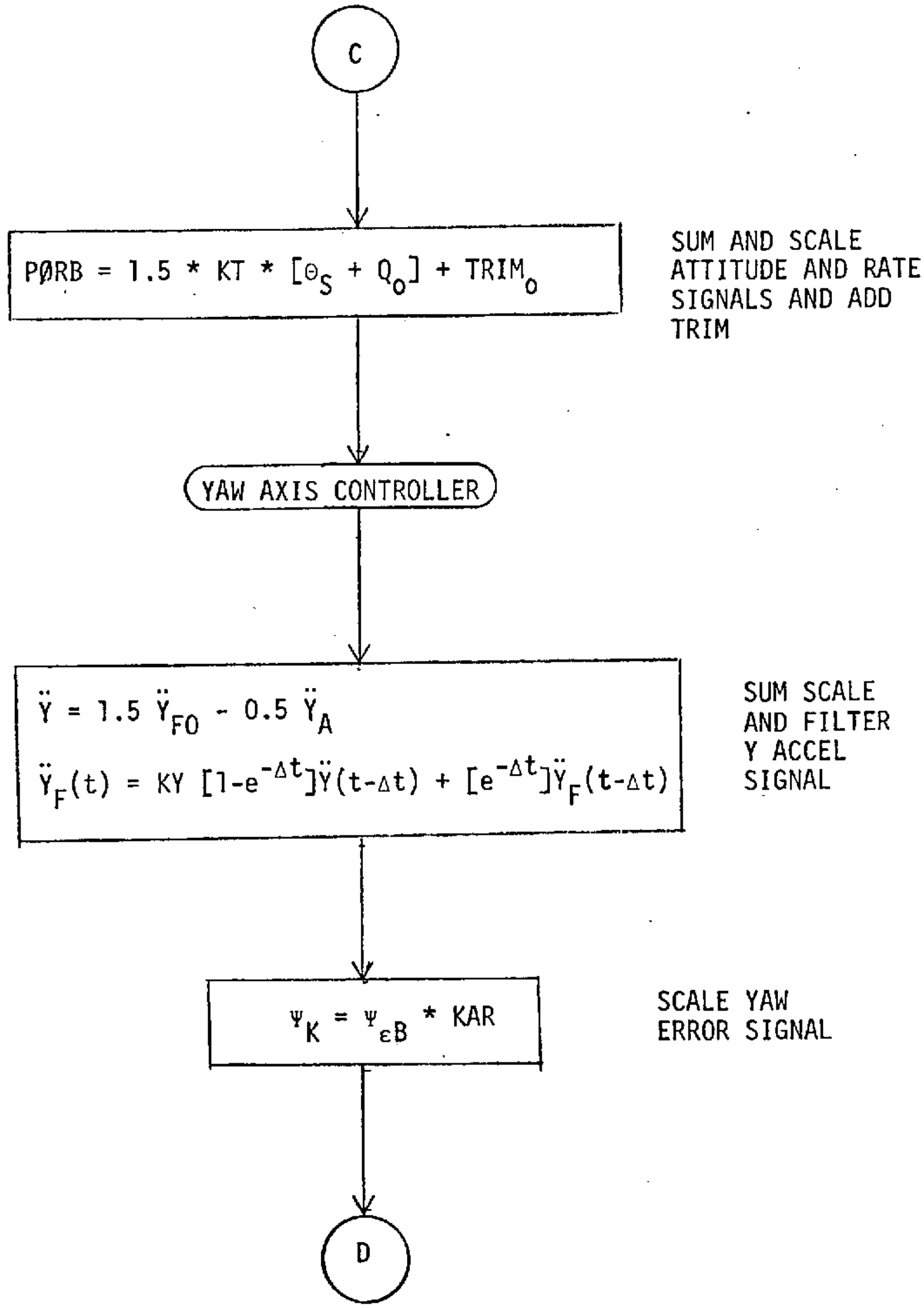
B

B

PITCH AXIS CONTROLLER

$\ddot{Z}(t) = 1.5\ \ddot{Z}_{FO} + 0.5\ \ddot{Z}_A - \ddot{Z}_C$ $\ddot{Z}_F(t) = KZ\ [1 - e^{-\Delta t}]\ddot{Z}(t-\Delta t) + [e^{-\Delta t}]\ddot{Z}_F(t-\Delta t)$	SUM, SCALE AND FILTER Z ACCEL SIGNAL
$\Theta_K = \Theta_{\varepsilon B} * KAQ$	SCALE PITCH ERROR SIGNAL
$\Theta_S = \Theta_K - \ddot{Z}_F - \gamma_{CI} * COS\ \phi_B$	SUM ATTITUDE, ACCEL AND RATE SIGNALS
$Q_K = 0.5\ [Q_4 + Q_5]$	SUM AND SCALE SRB RATE GYRO SIGNALS
$PSRM = 1.1 * KT * [\Theta_S + Q_K] + TRIM_S$	SUM AND SCALE ATTITUDE AND RATE SIGNAL AND ADD TRIM

C


C
$PØRB = 1.5 * KT * [\Theta_S + Q_o] + TRIM_o$
SUM AND SCALE ATTITUDE AND RATE SIGNALS AND ADD TRIM
YAW AXIS CONTROLLER
$\ddot{Y} = 1.5\ \ddot{Y}_{FO} - 0.5\ \ddot{Y}_A$
$\ddot{Y}_F(t) = KY\ [1-e^{-\Delta t}]\ddot{Y}(t-\Delta t) + [e^{-\Delta t}]\ddot{Y}_F(t-\Delta t)$
SUM SCALE AND FILTER Y ACCEL SIGNAL
$\Psi_K = \Psi_{\varepsilon B} * KAR$
SCALE YAW ERROR SIGNAL
D

D

$$RA(t) = KY * (-7.06) * \dot{\phi}_B$$

$$RA_F(t) = RA(t) - RA(t-\Delta t) + RA_F(t-\Delta t)[e^{-\Delta t}]$$

SCALE AND FILTER
ROLL RATE SIGNAL

$$\Psi_S = \Psi_K + \ddot{\Psi}_F + \dot{\gamma}_{CI} * SIN\ \phi_B + RA_F(t)$$

SUM AND SCALE
YAW ATTITUDE,
RATE ACCEL AND
ROLL RATE CROSS
FEED SIGNALS

$$YSRBL = 1.4 * KT * [R_4 + \Psi_S]$$

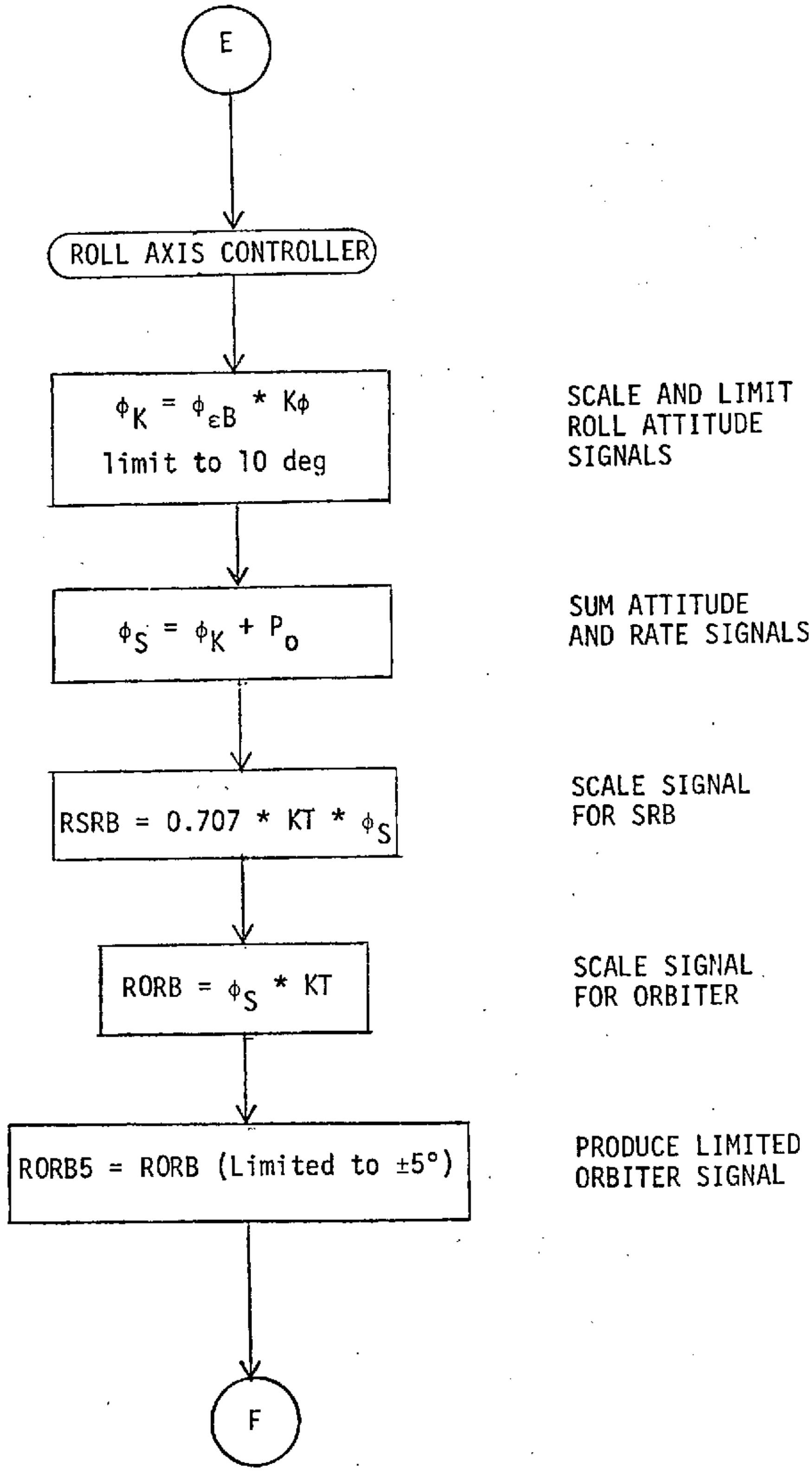
$$YSRBR = 1.4 * KT * [R_S + \Psi_S]$$

SUM AND SCALE
ATTITUDE AND
RATE SIGNALS
FOR SRB

$$YORB = 2 * KT * R_o$$

SUM AND SCALE
ATTITUDE AND
RATE SIGNALS
FOR ORBITER

E


E
ROLL AXIS CONTROLLER
$\phi_K = \phi_{\epsilon B} * K\phi$
limit to 10 deg
SCALE AND LIMIT ROLL ATTITUDE SIGNALS
$\phi_S = \phi_K + P_o$
SUM ATTITUDE AND RATE SIGNALS
$RSRB = 0.707 * KT * \phi_S$
SCALE SIGNAL FOR SRB
$RORB = \phi_S * KT$
SCALE SIGNAL FOR ORBITER
RORB5 = RORB (Limited to ±5°)
PRODUCE LIMITED ORBITER SIGNAL
F

F

MIXING LOGIC

$$\delta_{oP1} = AOP(1) * PORB + RP(1) * RORB5 + TRIM_o$$

$$\delta_{oP2} = AOP(2) * PORB + RP(2) * RORB5 + TRIM_o$$

$$\delta_{oP3} = AOP(3) * PORB + RP(3) * RORB5 + TRIM_o$$

$$\delta_{oY1} = AOY(1) * YORB + RY(1) * RORB$$

$$\delta_{oY2} = AOY(2) * YORB + RY(2) * RORB$$

$$\delta_{oY3} = AOY(3) * YORB + RY(3) * RORB$$

MIX SIGNALS TO GENERATE
INDIVIDUAL ORBITER
ENGINE COMMANDS

G

G

$$DELP4 = AOP(4) * PSRB + RP(4) * RSRM + TRIM_S$$

$$DELP5 = AOP(5) * PSRB + RP(5) * RSRM + TRIM_S$$

$$DELY4 = AOY(4) \; (\; YSRM + RY(4) * RSRM$$

$$DELY5 = AOY(5) \; (\; YSRM + RY(5) * RSRM$$

MIX SIGNALS TO GENERATE
INDIVIDUAL SRB ENGINE
COMMANDS ASSUMING ACTUATOR
ORIENTATION IS 0° - 90°

$$\delta_{SP4} = DELP4 * COS\ 45° - DELY4 * SIN\ 45°$$

$$\delta_{SP5} = DELP5 * COS\ 45° - DELY5 * SIN\ 45°$$

$$\delta_{SY4} = DELP4 * SIN\ 45° + DELY4 * COS\ 45°$$

$$\delta_{SY5} = DELP5 * SIN\ 45° + DELY5 * COS\ 45°$$

TRANSFORM SIGNALS TO
GENERATE SRB ENGINE
COMMANDS FOR 45° - 135"
ACTUATOR ORIENTATION

H

H

$$\delta_{OMS_{P1}} = AOP(6) * PORB + RP(6) * RORB5$$

$$\delta_{OMS_{P2}} = AOP(7) * PORB + RP(7) * RORB5$$

$$\delta_{OMS_{Y1}} = AOY(6) * YORB + RY(6) * RORB$$

$$\delta_{OMS_{Y2}} = AOY(7) * YORB + RY(7) * RORB$$

MIX SIGNALS TO GENERATE OMS ENGINE CMDS

RUDDER = RR(2) * RORB + AOY(8) * YORB

GENERATE RUDDER COMMAND

2.5.3 Nomenclature

AOP(i)	Mixing Logic Coefficients for Pitch Actuators Used for Pitch Control, i = 1-8
AOY(i)	Mixing Logic Coefficients for Yaw Actuators Used for Yaw Control, i = 1-8
K	Roll Command Filter Constant
KAQ	Pitch Attitude Error Gain
KAR	Yaw Attitude Gain
KT	Rate Gain
KY	Lateral Accelerometer Gain
KZ	Normal Accelerometer Gain
$K\phi$	Roll Attitude Gain
P_o	Orbiter Roll Rate Gyro Signal
PORB	Pitch Signal to Orbiter Engines
PSRB	Pitch Signal to SRB Engines
Q_K	Summed SRB Pitch Rate Signal
Q_o	Orbiter Pitch Rate Gyro Signal
Q_4	Left SRB Pitch Rate Gyro Signal
Q_5	Right SRB Pitch Rate Gyro Signal
R_o	Orbiter Yaw Rate Gyro Signal
R_4	Left SRB Yaw Rate Gyro Signal
R_5	Right SRB Yaw Rate Gyro Signal
RA	Roll Rate Crossfeed Signal
RA_F	Filtered Roll Rate Crossfeed Signal
RORB	Roll Signal to Orbiter Engines
RORB5	Roll Signal to Orbiter Engines Limited to ±5 degrees

RP(i)	Mixing Logic Coefficients for Pitch Actuators Used for Roll Control, i = 1-7
RR(i)	Mixing Logic Coefficients for Aero Actuators Used for Roll Control, i = 1-2
RSRB	Roll Signal to SRB Engines
RUDDER	Deflection Command to Rudder
RY(i)	Mixing Logic Coefficients for Yaw Actuators Used for Roll Control, i = 1-7
$TRIM_O$	Trim Signal to Orboter Pitch Signal
$TRIM_S$	Trim Signal to SRB Pitch Signal
$\ddot{Y}$	Summed Lateral Accelerometer Signal
$\ddot{Y}_A$	Aft Lateral Accelerometer Signal
$\ddot{Y}_F$	Filtered and Scaled Lateral Accelerometer Signal
$\ddot{Y}_{FO}$	Forward Lateral Accelerometer Signal
YORB	Yaw Signal to Orbiter Engines
YSRBL	Yaw Signal to Left SRB Engine
YSRBR	Yaw Signal to Right SRB Engine
$\ddot{Z}$	Summed Normal Accelerometer
$\ddot{Z}_A$	Aft Normal Accelerometer Signal
$\ddot{Z}_C$	Normal Accelerometer Command
$\ddot{Z}_F$	Filtered and Scaled Normal Accelerometer Signal
$\ddot{Z}_{FO}$	Forward Normal Accelerometer Signal
$\dot{\gamma}_{CI}$	Inertial Flight Path Angle Rate Command
ϕ_{CI}	Inertial Roll Attitude Command
ϕ_{CIF}	Filtered Inertial Roll Attitude Command

$\phi_{\varepsilon B}$	Body Roll Error
$\phi_{\varepsilon I}$	Inertial Roll Error
ϕ_I	Inertial Roll Attitude
ϕ_K	Scaled Roll Attitude Signal
ϕ_S	Summed Roll Attitude and Rate Signal
$\dot{\phi}_K$	Roll Attitude Signal Rate of Change
Θ_{CI}	Inertial Pitch Attitude Command
$\Theta_{\varepsilon B}$	Body Pitch Attitude Error
$\Theta_{\varepsilon I}$	Inertial Attitude Error
Θ_I	Inertial Pitch Attitude
Θ_K	Scaled Pitch Attitude Signal
Θ_S	Summed Pitch Attitude and Rate Signal
Θ_{CB}	Body Pitch Rate Command
$\Psi_{\varepsilon B}$	Body Yaw Attitude Error
$\Psi_{\varepsilon I}$	Inertial Yaw Attitude Error
Ψ_I	Inertial Yaw Attitude
Ψ_{CB}	Body Yaw Rate Command
Ψ_K	Scaled Yaw Attitude Signal
Ψ_S	Summed Yaw Attitude and Rate Signal
δ_{oP1}	Deflection Command to Pitch Actuator of Orbiter Engine #1
δ_{oP2}	Deflection Command to Pitch Actuator of Orbiter Engine #2
δ_{oP3}	Deflection Command to Pitch Actuator of Orbiter Engine #3
δ_{SR5}	Deflection Command to Right Actuator of Left SRB
δ_{SR6}	Deflection Command to Right Actuator of Right SRB
δ_{oY1}	Deflection Command to Yaw Actuator of Orbiter Engine #1
δ_{oY2}	Deflection Command to Yaw Actuator of Orbiter Engine #2

δ_{oY3}	Deflection Command to Yaw Actuator of Orbiter Engine #3
δ_{SL5}	Deflection Command to Left Actuator of Left SRB
δ_{SL6}	Deflection Command to Left Actuator of Right SRB
$\delta_{OMS_{P1}}$	Deflection Command to Pitch Actuator of OMS Engine #1
$\delta_{OMS_{P2}}$	Deflection Command to Pitch Actuator of OMS Engine #2
$\delta_{OMS_{Y1}}$	Deflection Command to Yaw Actuator of OMS Engine #1
$\delta_{OMS_{Y2}}$	Deflection Command to Yaw Actuator of OMS Engine #2
Δt	Computation Frequency for Control Subroutine

2.5.4 Input/Output

Inputs to this model are inertial attitude angles, body rotational rates, Y and Z translational acceleration, prestored engine trim deflection commands, prestored acceleration commands, and attitude commands from guidance. Outputs from this model are engine gimbal and aero surface deflection commands output via the CMDFIL routine.

2.6 CGAINS (Control Gains Equations)

2.6.1 Program Description

The CGAINS program is used to calculate the control gains necessary for a desired type of control during the Shuttle boost. There are several options for the control gains that are calculated: load minimum, drift minimum, or attitude control for the pitch and yaw gains; and thrust vector control or aero control for the roll gains. The following model presents the equations necessary to calculate these gains.

2.6.2 Math Model

The following quantities must be calculated each time the control gains are needed. The symbols used in these equations are defined in Table I.

$$l_{1p} = \bar{c}\,(C_{m\alpha}/C_{z\alpha}) + X_{cg} - X_R$$

$$l_{1y} = (\bar{b}\,C_{n\beta}/C_{y\beta}) + X_{cg} - X_R$$

$$N'_p = q\,S\,C_{z\alpha}$$

$$N'_y = q\,S\,C_{y\beta}$$

$$l_{ap} = X_{ACEL} - X_{cg}$$

$$l_{ay} = -l_{ap}$$

$$K_{1p} = [\sum_{i=1}^{7} F_{X_i}]/m$$

$$K_{2p} = N'_p/m$$

$$\ell_{X_i} = X_{cg} - X_{e_i}$$

$$\ell_{Y_i} = Y_{cg} - Y_{e_i}$$

$$Zi = Z_{cg} - Z_{e_i}$$

$$K_{3p} = -\{[\sum_{i=1}^{7} F_{X_i} * AOP_i] + (qSC_{3_{\delta_e}} AOP_8)\} / m$$

$$K_{1y} = -K_{1p}$$

$$K_{2y} = -N'_y / m$$

$$K_{3y} = \{[\sum_{i=1}^{7} F_{X_i} * AOY_i] + (qSCy_{\delta_r} AOY_8)\} / m$$

$$C_{1p} = -\ell_{1p} * N'_p / I_y$$

$$C_{ZP} = \{[\sum_{i=1}^{7} F_{X_i} * AOP_i + \ell_{X_i}] + (qS\bar{c}C_{m_{\delta_e}} AOP_8)\} / I_y$$

$$\Delta_p = C_{2p} K_{2p} - C_{1p} K_{3p}$$

$$C_{1y} = -l_{1y} N'_y / I_Z$$

$$C_{2y} = \{[\sum_{i=1}^{7} F_{X_i} * AOY_i * \ell_{X_i}] + qS\bar{b}C_{n_{\delta_r}} AOY_8\} / I_Z$$

$$\Delta_y = C_{2y} K_{2y} - C_{1y} K_{3y}$$

Pitch and Yaw Control Gains for Load Minimum Option

$$a_{0p} = 0.$$

$$g_{2p} = \frac{\omega_y^2 - C_{1p}}{\Delta_p + \omega_y^2 (K_{3p} + l_{ap} C_{2p})}$$

$$a_{1p} = \frac{2 \zeta_p \omega_y}{C_{2p}} \left[1 - g_{2p} (K_{3p} + l_{ap} C_{2p}) \right]$$

$$a_{0y} = 0.$$

$$g_{2y} = \frac{\omega_z^2 - C_{1y}}{\Delta_y + \omega_z^2 (K_{3y} + l_{ay} C_{2y})}$$

$$a_{1y} = \frac{2 \zeta_y \omega_z}{C_{2y}} \left[1 - g_{2y} (K_{3y} + l_{ay} C_{2y}) \right]$$

Pitch and Yaw Control Gains for Drift Minimum Option

$$g_{2p} = \frac{\omega_y^2 - (1 + C_{2p} K_{1p}/\Delta_p) C_{1p}}{C_{2p} K_{1p} + \Delta_p + \omega_y^2 (K_{3p} + l_{ap} C_{2p})}$$

$$a_{0p} = g_{2p} K_{1p} + C_{1p} K_{1p}/\Delta_p$$

$$a_{1p} = \frac{2 \zeta_p \omega_y}{C_{2p}} \left[1 - g_{2p} (K_{3p} + l_{ap} C_{2p}) \right]$$

$$g_{2y} = \frac{\omega_z^2 - (1 + C_{2y} K_{1y}/\Delta_y) C_{1y}}{C_{2y} K_{1y} + \Delta_y + \omega_z^2 (K_{3y} + l_{ay} C_{2y})}$$

$$a_{0y} = g_{2y} K_{1y} + C_{1y} K_{1y}/\Delta_y$$

$$a_{1y} = \frac{2 \zeta_y \omega_z}{C_{2y}} \left[1 - g_{2y} (K_{3y} + l_{ay} C_{2y}) \right]$$

$$g_{2p} = 0$$

$$a_{0p} = \frac{\omega_y^{2} - C_{1p}}{C_{2p}}$$

$$a_{1p} = \frac{2\ \zeta_p\ \omega_y}{C_{2p}}$$

$$g_{2y} = 0$$

$$a_{0y} = \frac{\omega_z^{2} - C_{1y}}{C_{2y} + N''}$$

$$a_{1y} = \frac{2\ \zeta_y\ \omega_z}{C_{2y} + N''}$$

Thrust Vector Control

$$C_{2r} = -[\sum_{i=1}^{6} (F_{X_i} * RP_i * \ell_{y_i} + F_{X_i} * RY_i + \ell_{z_i})] / I_X$$

$$a_{0r} = \frac{\omega_x^2}{C_{2r}}$$

$$a_{1r} = \frac{2\ \zeta_r\ \omega_x}{C_{2r}}$$

Aero Control

$$C_{2r} = -q S \bar{b}\ [C_{\ell_{\delta_a}}\ RR_1 - 1\ C_{\ell_{\delta_r}}\ RR_2] / I_X$$

$$a_{0r} = \frac{\omega_x^2}{C_{2r}}$$

$$a_{1r} = \frac{2\ \zeta_r\ \omega_x}{C_{2r}}$$

TVC and Aero Control

$$C_{2r} = -\{[\sum_{i=1}^{6} (F_{X_i} * RP_i * \ell_{y_i} + F_{X_i} * RY_i * \ell_{z_i})]$$

$$+ q S \bar{b}\ [C_{\ell_{\delta_a}}\ RR_1 + C_{\ell_{\delta_r}}\ RR_2]\ \} / I_X$$

a_{0r} and a_{1r} same as above.

2.6.3 Nomenclature

Variable	Definition
AOP_i	Mixing Logic Coefficients for Pitch Actuators Used for Pitch Control, i = 1-8
AOY_i	Mixing Logic Coefficients for Yaw Actuators Used for Yaw Control, i = 1-8
a_{op}	Pitch Attitude Gain
a_{oy}	Yaw Attitude Gain
a_{1p}	Pitch Attitude Rate Gain
a_{1y}	Yaw Attitude Rate Gain
$\bar{b}$	Wing Span
C	Coefficients for Calculation of Gain
$\bar{c}$	Mean Aerodynamic Chord
$C_{\ell_{\delta_a}}$	Aero Rolling Moment Coefficient Due to Aileron Deflection
$C_{\ell_{\delta_r}}$	Aero Rolling Moment Coefficient Due to Rudder Deflection
Cm_{α}	Aero Pitching Moment Coefficient Due to Angle-of-Attack
Cm_{δ_e}	Aero Pitching Moment Coefficient Due to Elevator Deflection
Cn_{β}	Aero Yawing Moment Coefficient Due to Sideslip-Angle
Cn_{δ_r}	Aero Yawing Moment Coefficient Due to Rudder Deflection
Cy_{β}	Aero Side Force Coefficient Due to Sideslip-Angle
Cy_{δ_r}	Aero Side Force Coefficient Due to Rudder Deflection

Variable	Definition
C_{z_α}	Aero Normal Force Coefficient Due to Angle-of-Attack
$C_{z_{\delta_e}}$	Aero Normal Force Coefficient Due to Elevator Deflection
F_{X_i}	Thrust in X direction from engine i
g_{2p}	Pitch Acceleration Gain
g_{2y}	Yaw Acceleration Gain
I	Moments of Inertia
K	Coefficients for Calculation of Gains
l	Moment Arms
m	Mass
N'	Partial of Normal Force
q	Dynamic Pressure
RP_i	Mixing Logic Coefficients for Pitch Actuators Used for Roll Control, i = 1-7
RR_i	Mixing Logic Coefficients for Aero Actuators Used for Roll Control, i = 1-2
RY_i	Mixing Logic Coefficients for Yaw Actuators Used for Roll Control, i = 1-7
S	Aero Reference Area
X_{ACCEL}	X Accelerometer Location
X_{CG}, Y_{CG}, Z_{CG}	Location of Center of Gravity
X_R, Y_R, Z_R	Location of Aero Reference Point
X_{e_i}, Y_{e_i}, Z_{e_i}	Engine Locations, i = 1-6

Variable	Definition
Δ_p, Δ_y	Temporary Variable
ζ	Damping Ratio
ω	Natural Frequency

2.6.4 Input/Output

The control gains routine calculates the attitude gains, attitude rate gains and accelerometer gains for each of the desired conditions which have been previously mentioned. These gains are also calculated for a range of natural frequencies (ω). These results are output on a scratch tape or FASTRAND file for processing by a plotting program.

2.7 FLTSEQ (Flight Sequencing)

2.7.1 Program Description

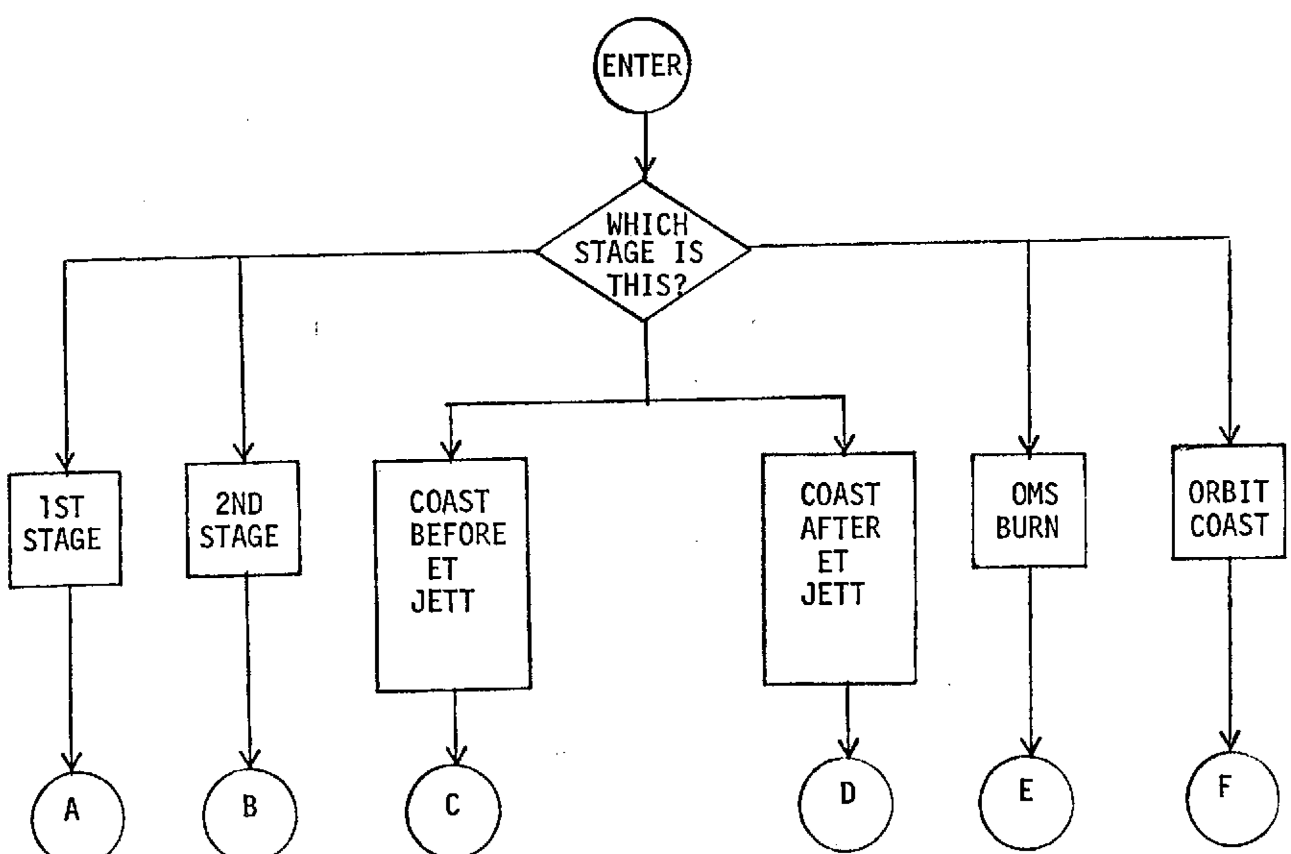
Although not currently active, this program is called by the Flight Control System (BLC) and is used to initiate staging and provide flight sequencing logic as required.

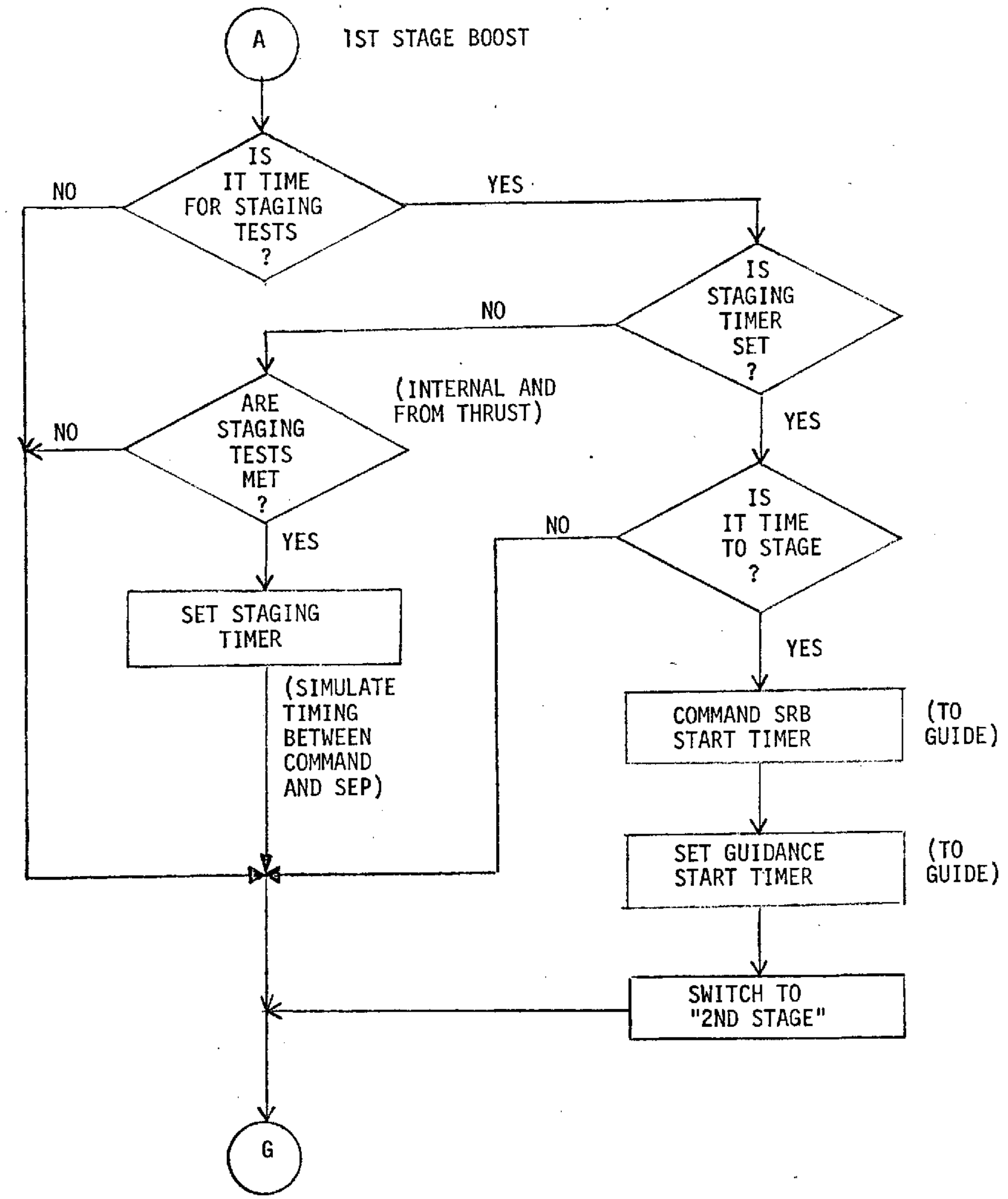
The switching points between stages were defined by the following events: 1) SRB separation and guidance initiation; 2) main engines shutdown and RCS activation; 3) external tank jettison; 4) OMS burn initiation and RCS de-activation; and 5) OMS shutdown and RCS actuation at orbit insertion.

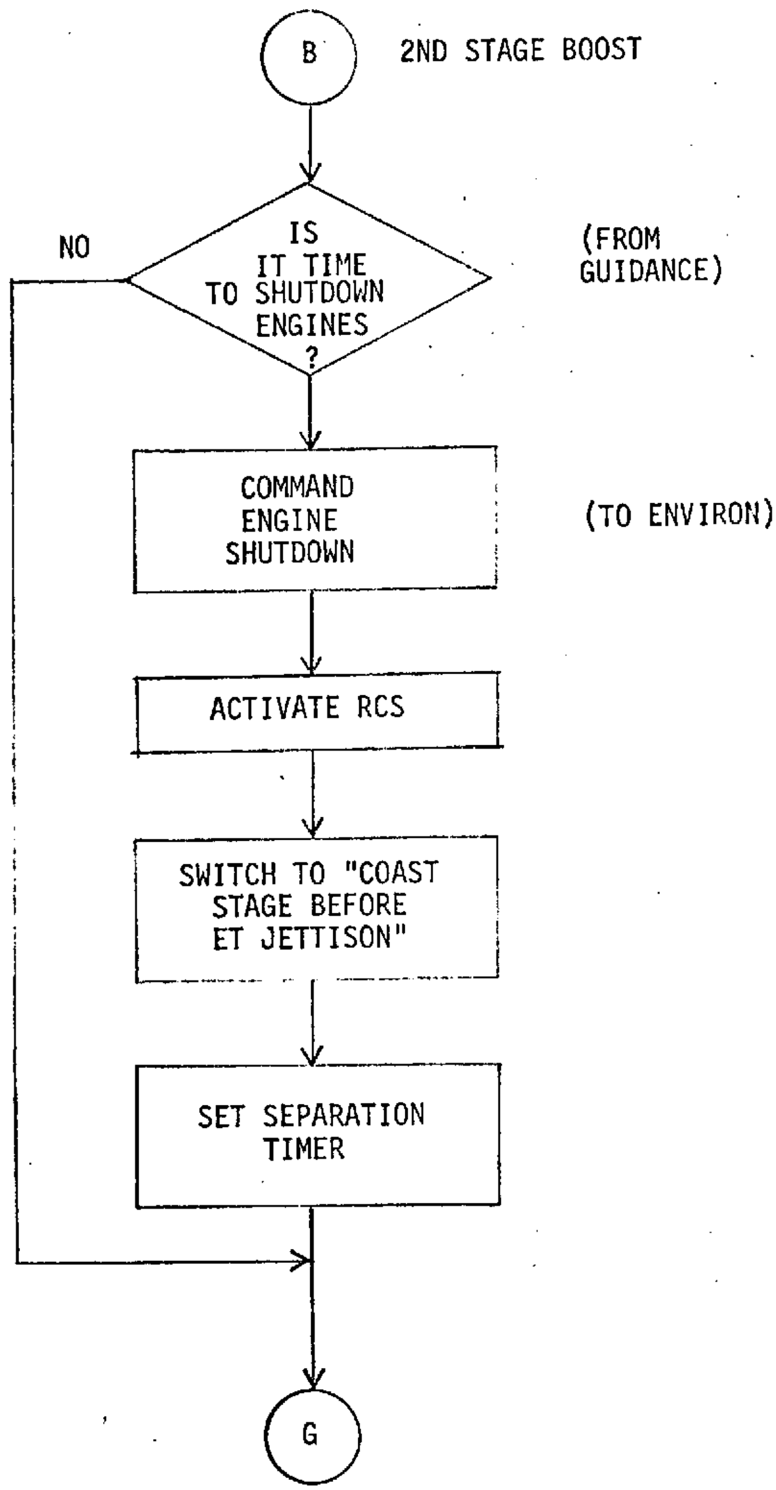
2.7.2 Math Model

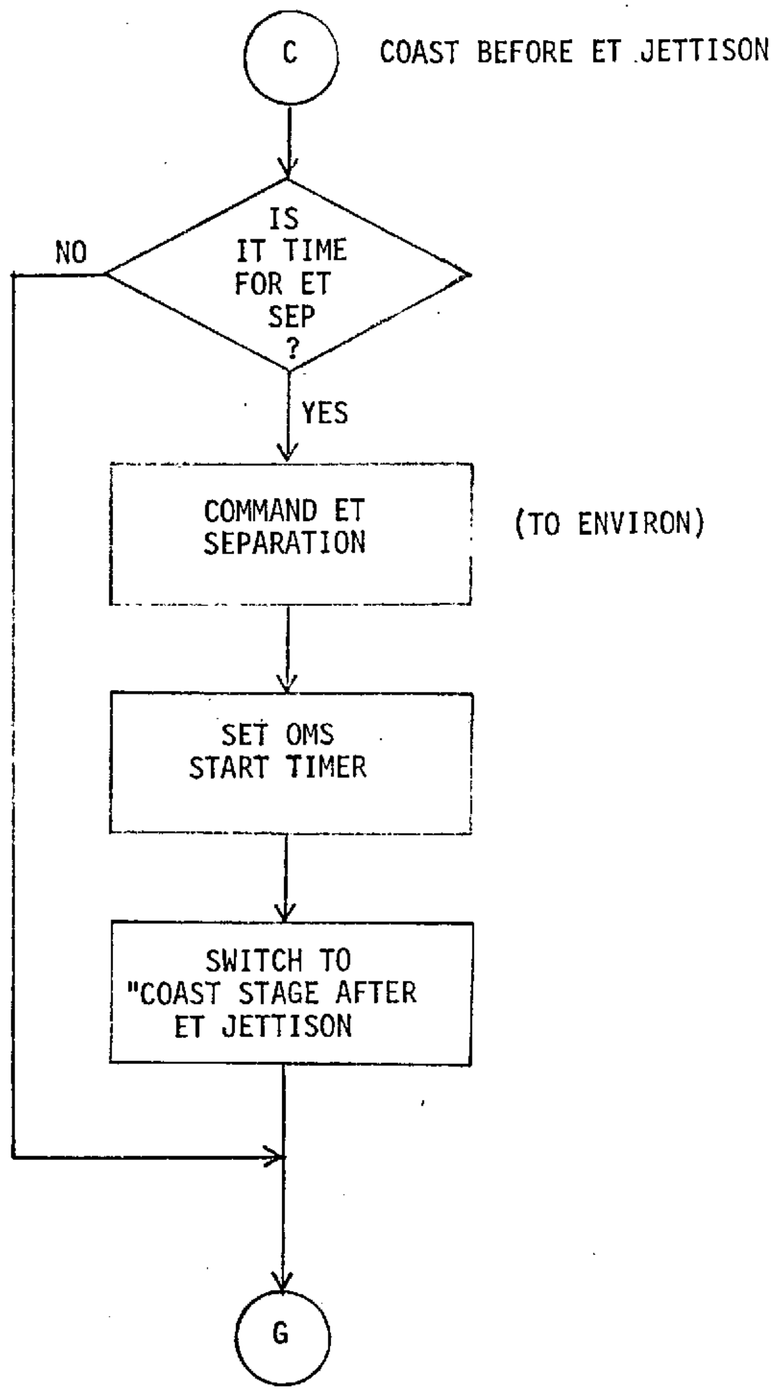
BOOST TO ORBIT

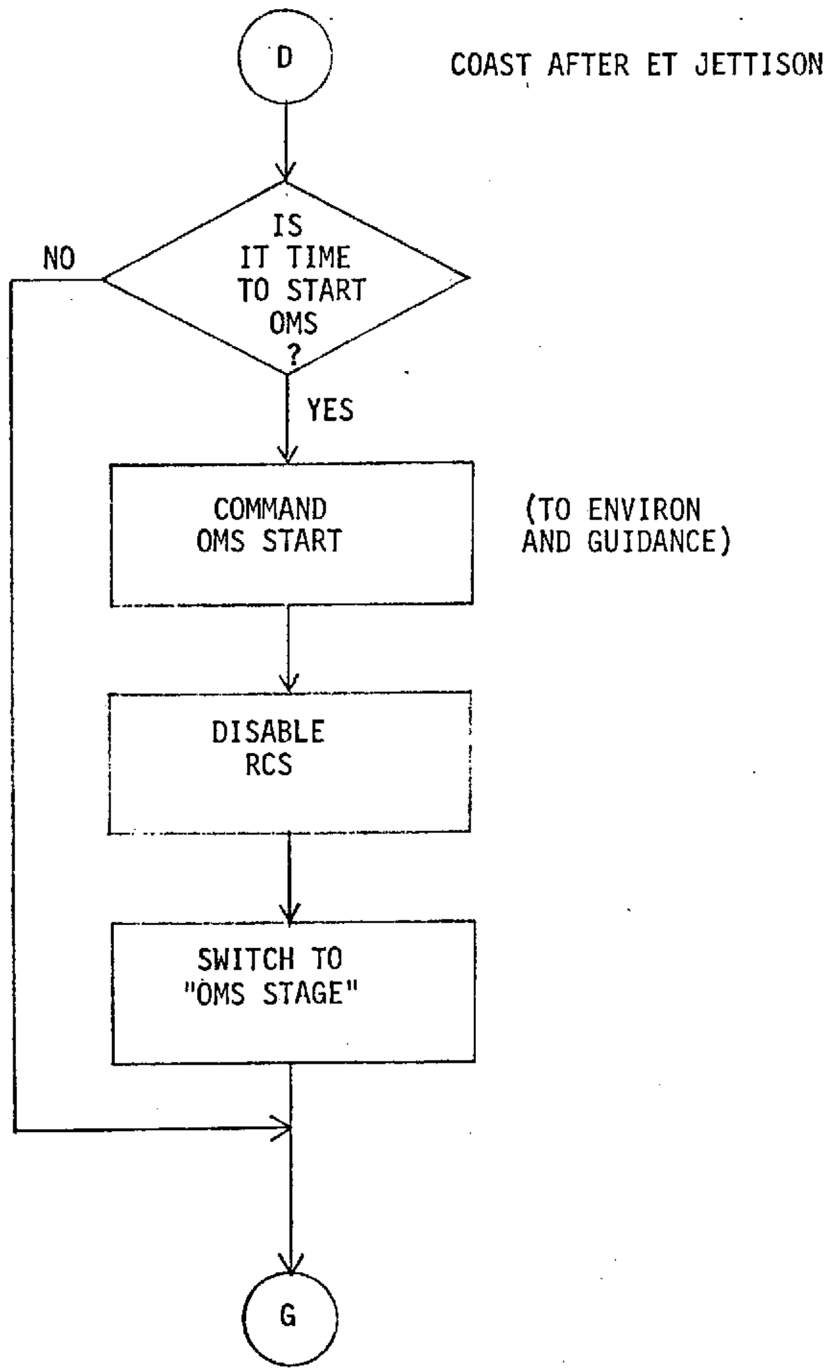
FLIGHT CONTROL LOGIC

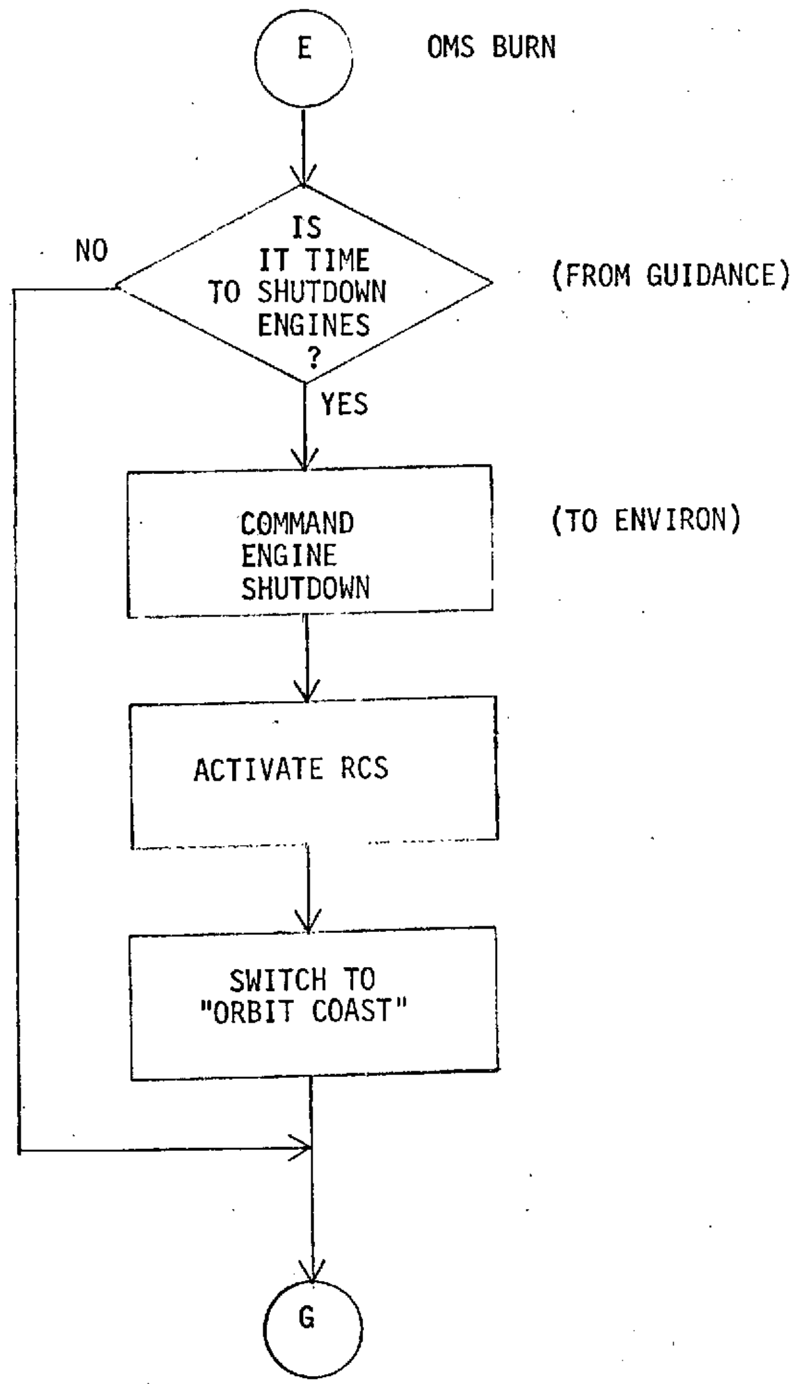



A
1ST STAGE BOOST
IS IT TIME FOR STAGING TESTS ?
NO
YES
IS STAGING TIMER SET ?
NO
ARE STAGING TESTS MET ?
(INTERNAL AND FROM THRUST)
NO
YES
YES
IS IT TIME TO STAGE ?
NO
SET STAGING TIMER
YES
(SIMULATE TIMING BETWEEN COMMAND AND SEP)
COMMAND SRB START TIMER
(TO GUIDE)
SET GUIDANCE START TIMER
(TO GUIDE)
SWITCH TO "2ND STAGE"
G


B
2ND STAGE BOOST
NO
IS IT TIME TO SHUTDOWN ENGINES ?
(FROM GUIDANCE)
COMMAND ENGINE SHUTDOWN
(TO ENVIRON)
ACTIVATE RCS
SWITCH TO "COAST STAGE BEFORE ET JETTISON"
SET SEPARATION TIMER
G


C
COAST BEFORE ET JETTISON
IS IT TIME FOR ET SEP ?
NO
YES
COMMAND ET SEPARATION
(TO ENVIRON)
SET OMS START TIMER
SWITCH TO "COAST STAGE AFTER ET JETTISON
G


D
COAST AFTER ET JETTISON
IS IT TIME TO START OMS ?
NO
YES
COMMAND OMS START
(TO ENVIRON AND GUIDANCE)
DISABLE RCS
SWITCH TO "OMS STAGE"
G


E
OMS BURN
IS IT TIME TO SHUTDOWN ENGINES ?
NO
(FROM GUIDANCE)
YES
COMMAND ENGINE SHUTDOWN
(TO ENVIRON)
ACTIVATE RCS
SWITCH TO "ORBIT COAST"
G

2.8 GUIDE (Guidance Model)

Due to recent revisions and updates to the guidance equations, these models are not shown here but are presented in Volume V of this report.

2.9 MASPRO (Mass Properties)

2.9.1 Program Description

This model provides the mass properties, which consist of center of gravity travel, moments of inertia as a function of weight and total mass calculation.

2.9.2 Math Model

$W = W - \dot{W}$

At Staging $W = W - W\ (I)$

CG_x = table lookup $F(W)$

CG_y = table lookup $F(W)$

CG_z = table lookup $F(W)$

I_{xx} = table lookup $F(W)$

I_{yy} = table lookup $F(W)$

I_{zz} = table lookup $F(W)$

I_{xz} = table lookup $F(W)$

2.9.3 Nomenclature

W = total vehicle weight

$W(1)$ = solid rocket booster weight

$W(2)$ = external tank weight

$\dot{W}$ = weight flow rate input from THRUST

CG_x = X center of gravity location from vehicle reference

CG_y = Y center of gravity location from vehicle reference

CG_z = Z center of gravity location from vehicle reference

I_{xx} = Mass moment of inertia about X axis

I_{yy} = Mass moment of inertia about Y axis

I_{zz} = X - Z cross product of inertia

2.9.4 Input/Output

Input: $\dot{W}$ from THRUST

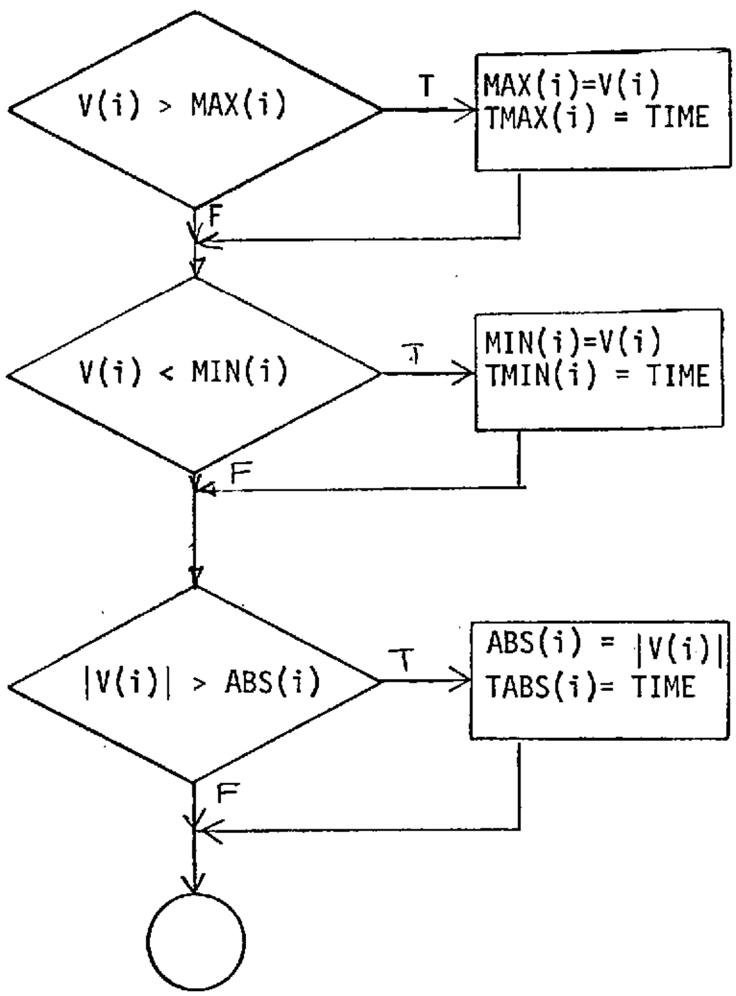
Output: W, CG's and I's.

2.10 MAXMIN (Max and Min Parameter Dump)

2.10.1 Program Description

This program is used to collect the extreme values of certain flight parameters. At the stop time scheduled for MAXMIN the collected values are printed. This routine may be scheduled as often as desired, as long as the calls do not overlap.

2.10.2 Math Model



2.10.3 Nomenclature

V(i)	Parameter value to be tested for magnitude
MAX(i)	Maximum value of parameter i collected
MIN(i)	Minimum value of parameter i collected
ABS(i)	Absolute maximum value of parameter i collected
TMAX(i)	Time of occurrence of maximum
TMIN(i)	Time of occurrence of minimum
TABS(i)	Time of occurrence of absolute maximum

2.11 ORBIT1 (Orbit Parameters)

2.11.1 Program Description

This math model calculates the parameters of the trajectory achieved at insertion from knowledge of the state vector in polar equatorial coordinates at the time of orbiter engine shutdown. The program calculates node, inclination angle, orbit phase angle, eccentricity, orbit parameter, true anomaly, apogee altitude, and perigee altitude.

2.11.2 Math Model

$$\eta = \text{Arctan}\left(\frac{-V_{YPE}\,R_{ZPE} + V_{ZPE}\,R_{YPE}}{-V_{XPE}\,R_{ZPE} + V_{ZPE}\,R_{XPE}}\right)$$

$$\zeta = \text{Arctan}\left(\frac{V_{ZPE}}{-V_{XPE}\sin\eta + V_{YPE}\cos\eta}\right)$$

$$\nu = \text{Arctan}\left(\frac{-V_{XPE}\cos\eta - V_{YPE}\sin\eta}{\cos\zeta(-V_{XPE}\sin\eta + V_{YPE}\cos\eta) + V_{ZPE}\sin\zeta}\right)$$

$$R = \sqrt{R_{XPE}^{\;2} + R_{TPE}^{\;2} + R_{ZPE}^{\;2}}$$

$$V = \sqrt{V_{XPE}^{\;2} + V_{YPE}^{\;2} + V_{ZPE}^{\;2}}$$

$$P = \frac{|R \times V|^2}{K}$$

$$SMA = \frac{K}{\frac{2K}{R} - V^2}$$

$$\varepsilon = \sqrt{1 - \frac{P}{SMA}}$$

$$FP = \frac{R \cdot V}{\sqrt{R^2V^2 - |R \cdot V|^2}}$$

$$TA = \text{Arctan} \frac{(P)(FP)}{P-R}$$

$$R_p = SMA\ (1-E) - R_e$$

$$R_a = 2 \cdot SMA - R_p - 2\ R_e$$

2.11.3 Nomenclature

V_{XPE}, V_{YPE}, V_{ZPE} = Velocity in polar-inertial coordinates

R_{XPE}, R_{YPE}, R_{ZPE} = Position in polar-inertial coordinates

η = Longitude of the ascending node

ζ = Inclination angle

ε = Eccentricity

K = Gravitational constant = 1.407654×10^{16} ft^3/sec^2

ν = Orbit phase angle (angle between equator and perigee)

P = Semi-latus rectum

SMA = Semi-major axis

FP = Tangent of flight path angle

TA = True anomaly

R_p = Altitude at perigee

R_a = Altitude at apogee

R_e = Mean radius of the earth

2.11.4 Input/Output

The ORBITI math model requires the vehicle state vector in platform equatorial coordinates as input. As output the model prints out ascending node, inclination angle, orbit phase angle, eccentricity, orbit parameter, true anomaly, apogee altitude, and perigee altitude.

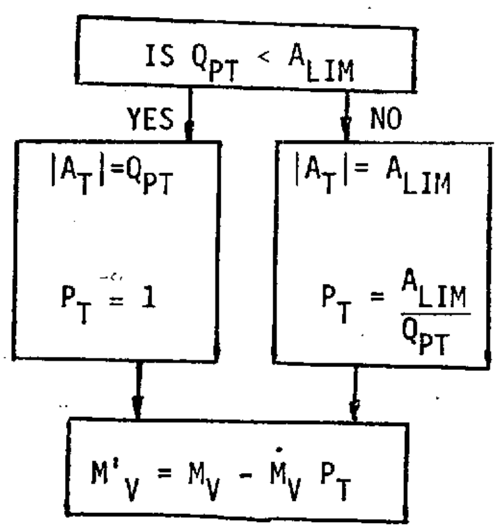
2.12 ORBITR (3D EQUATIONS OF MOTION)

2.12.1 Program Description

This Math Model calculates vehicle acceleration from active guidance commands and integrates to get velocity and position in platform coordinates. Polar-Equatorial and Local Vertical coordinates systems are erected to calculate latitude, longitude, and flight path angle. Logic is included for acceleration limiting, integration cycle time rectification, and velocity cutoff.

2.12.2 Math Model

$$Q_{PT} = \sum_{I=1}^{N} \frac{T_V(I)}{M_V}$$



$$\vec{Q}_{AP} = \frac{\vec{A}_P}{|A_P|}$$

$$\vec{A}_{PT}' = Q_{PT} \, P_T \, \vec{Q}_{AP} + \vec{A}_{GRAV}$$

$$\vec{V}_P' = \int_t \vec{A}_{PT}\, dt + \vec{V}_P$$

$$\vec{R}_P' = \int_t \vec{V}_P\, dt + \vec{R}_P$$

$$\vec{R}_F = [\alpha]^T\, \vec{R}_P'$$

$$\lambda_V = \text{Arcsin}\left(\frac{R_F(3)}{|R_F|}\right)$$

$$\phi = \text{Arctan}\left(\frac{R_F(2)}{R_F(1)}\right) - \omega_e\,(t_L+t)$$

$$\vec{V}_{LV} = [\delta]^T\, [\alpha]^T\, \vec{V}'_P$$

$$\gamma = \text{Arcsin}\left(\frac{V_{LV}(1)}{|V_{LV}|}\right)$$

Integration Cycle Time Rectification

Acceleration limiting will most likely occur between integration time points. Therefore the integration for this interval must be done in two parts.

Velocity Cutoff

The program will be terminated either when $|V_P|$ exceeds $\dot{Z}$ or when M_V is less than M_{VO}, whichever occurs first.

2.12.3 Nomenclature

Q_{PT} = Thrust acceleration at full throttle

N = Number of engines

T_V = Maximum vacuum thrust of main engine

M'_V = Current vehicle mass

A_{LIM} = Maximum allowed vehicle acceleration

$|A_T|$ = Magnitude of acceleration due to thrust

P_T = Throttle setting

$\dot{M}_V$ = Total mass flow rate for all engines at full throttle

$\vec{A}_P$ = Acceleration command from guidance in platform coordinates

$\vec{Q}_{AP}$ = Unit vector acceleration command

$\vec{A}_{PT}'$ = Current total vehicle acceleration in platform coordinates

$\vec{A}_{GRAV}$ = Acceleration due to gravity in platform coordinates

$\vec{V}_P$ = Velocity of vehicle in platform coordinates on last pass

$\vec{V}_P'$ = Current vehicle velocity

$\vec{R}_P$ = Position of vehicle in platform coordinates on last pass

$\vec{R}_P'$ = Current vehicle position

$\vec{R}_F$ = Position of vehicle in polar equatorial coordinates

λ_L^* = Geodetic latitude of launch site

ϕ_L^* = Longitude of launch site

ω_e = Rotation rate of earth

T_L = Time of launch

t = Elasped time since launch

M_V = Mass of vehicle on last pass

A_L = Launch azimuth

λ_V = Present vehicle latitude

ϕ' = Temporary variable

$[\alpha]$ = Transformation from polar equatorial to platform coordinates

$[\delta]$ = Transformation from local vertical to polar equatorial coordinates

ϕ = Present vehicle longitude

γ = Flight path angle

$\vec{V}_{LV}$ = Velocity in local vertical coordinates

$\dot{Z}$ = Target velocity in guidance coordinates

M_{VO} = Mass of empty orbiter

2.12.4 Input/Output

The targeting program provides $\dot{Z}$. The Active Guidance program provides $\vec{A}_P$. The gravity program provides $\vec{A}_{GRAV}$. The resulting quantities calculated by this model are λ_V, ϕ, γ, V_P, R_P, A_{PT}, M_V, and $\vec{V}_{LV}$. Flight software commands accepted by this model are $\vec{A}_P$. The Active Guidance, Targeting, and Gravity programs must be present to provide inputs to the model.

2.13 ORBTAR (Boost Orbit Insertion Targeting Model)

2.13.1 Program Description

The Targeting program is used in the flight software to describe the orbit plane with respect to the launch pad. Position, velocity, and a unit vector normal to the orbit plane at perigee are calculated from a knowledge of perigee and apogee altitudes, location of the launch pad, orbit inclination, and an orbit parameter.

2.13.2 Math Model

$$R_P = R_e + h_P$$

$$R_A = R_e + h_A$$

$$A = \frac{R_P + R_A}{2}$$

$$R = R_P$$

$$Y = 0$$

$$Z = \text{unconstrained}$$

$$\dot{R} = 0$$

$$\dot{Y} = 0$$

$$\dot{Z} = \sqrt{\mu \left(\frac{2}{R_P} - \frac{1}{A}\right)}$$

$$S_G = \sin(\lambda)\cos(\beta) + \cos(A_Z)\cos(\lambda)\sin(\beta)$$

$$C_L = \sin(A_Z)\cos(\lambda)$$

$$C_G = \sqrt{1 - S_G^2}$$

$$C_P = \frac{C_L}{C_G}$$

$$\alpha = \cos(\gamma)/C_G$$

$$\Delta = \text{Arcsin}(C_P) - \text{Arcsin}(\alpha)$$

$$U_Q(1) = -\sin(\beta)\sin(\Delta)$$

$$U_Q(2) = \cos(\Delta)$$

$$U_Q(3) = \cos(\beta)\sin(\Delta)$$

2.13.3 Nomenclature

R_P = distance from earth center of mass to periapsis

R_A = distance from earth center of mass to apoapsis

h_P = altitude at periapsis

h_A = altitude at apoapsis

A = semi-major axis

ε = eccentricity

R_e = radius of earth

R = radial distance at insertion

Y = cross-range distance at insertion

Z = downrange distance at insertion

$\dot{R}$ = radial rate at insertion

$\dot{Y}$ = lateral velocity at insertion

$\dot{Z}$ = downrange velocity at insertion

S_G, C_L, C_G, C_P, α, Δ = temporary variables

λ = latitude of launch pad

μ = universal gravitational constant

β = orbit parameter

A_Z = launch azimuth

γ = orbit inclination angle

$\vec{U}_Q$ = unit vector normal to desired orbit plane in platform coordinates (see Section IV)

2.13.4 Input/Output

This model requires R_e, h_p, h_a, λ , β, A_Z, and γ as input and calculates R, Y, Z, $\dot{R}$, $\dot{Y}$, $\dot{Z}$, and $\vec{U}_Q$. The model needs to be called only once per simulation.

2.14 RCS (Reaction Control System)

2.14.1 Program Description

This program calculates the moments and linear accelerations applied to the vehicle due to RCS thrust commands from the flight software.

2.14.2 Math Model

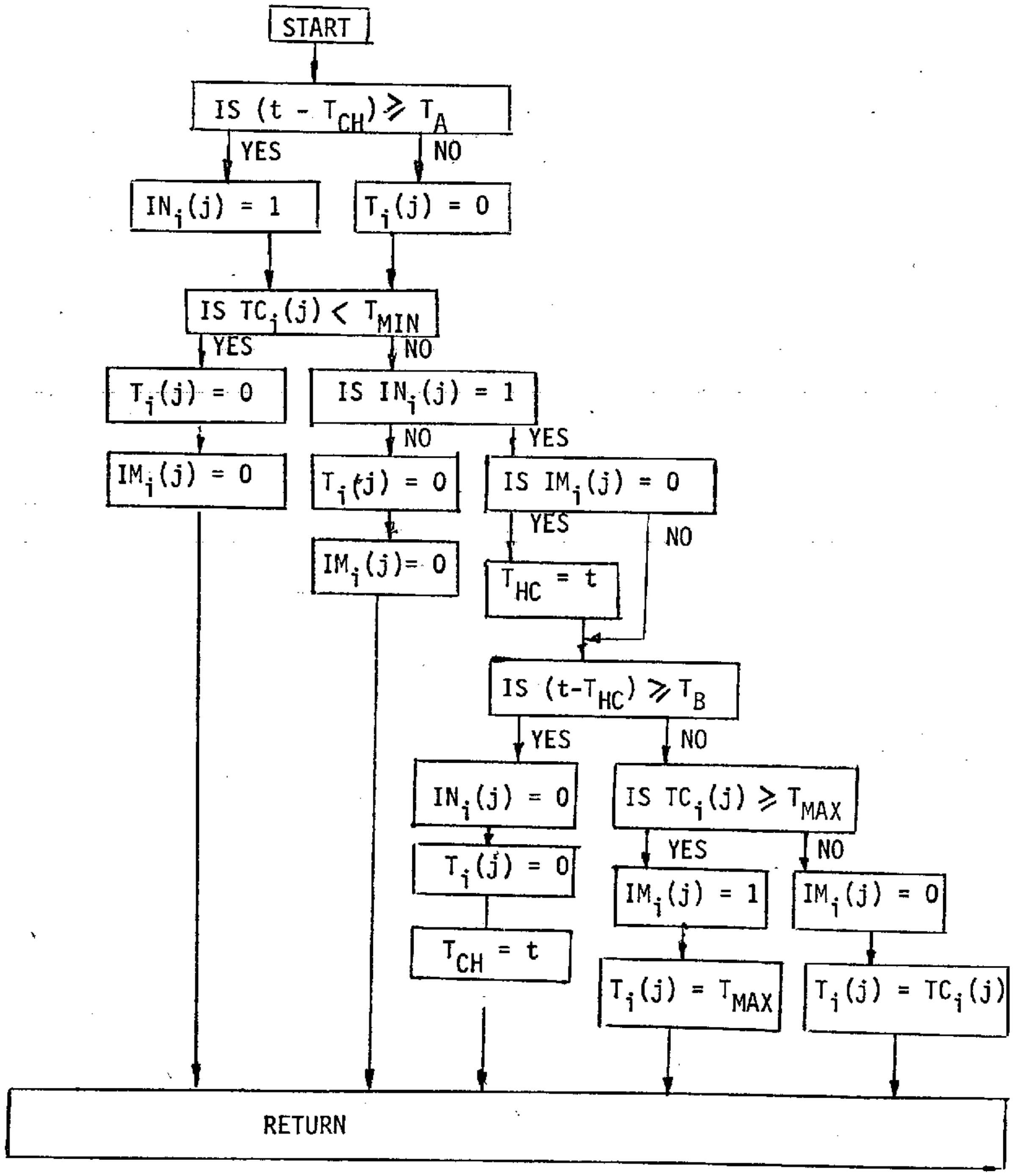
Thrust commands are conditioned according to the following limitations:

(1) A jet cannot be commanded to ignite unless the duration of ignition is some minimum value

(2) No jet can be ignited continuously longer than T_B seconds

(3) A jet cannot be commanded to ignite unless T_A seconds has elapsed since the previous ignition has ceased.

The following shows a logic diagram which can be used to implement these limitations.



RCS LOGIC DIAGRAM

FIGURE 2-1

2.14.3 Nomenclature

i = engine index

j = 1, 2, or 3 for x, y, or z respectively

T_{MAX} = thrust achieved with continuous ignition

T_{MIN} = average thrust for a computation cycle for minimum thrust duration

$TC_i(j)$ = commanded thrust

$T_i(j)$ = realized thrust

$IM_i(j)$ = indicator for continuous thrust for the previous pass (set equal to zero for restart)

$IN_i(j)$ = thrust enable flag (set equal to zero for restart)

T_{CH} = check time for T_A

T_{HC} = check time for T_B

t = current time at entry to RCS program

$$\vec{M}_i = \vec{P}_i \times \vec{T}_i \tag{1}$$

$$\begin{aligned} P_i(1) &= ELX_i - XCG \\ P_i(2) &= ELY_i - YCG \\ P_i(3) &= ELZ_i = ZCG \end{aligned} \tag{2}$$

$\vec{P}_i$ = engine position vector for the i^{th} jet; ELX_i, ELY_i, ELZ_i = X, Y, and Z locations, respectively, of the i^{th} RCS jet cluster; XCG, YCG, ZCG = X, Y, and Z locations, respectively of the vehicle center of mass

i = engine index

$\vec{M}_i$ = moment vector due to thrust from the i^{th} jet

$\vec{T}_i$ = thrust vector for the i^{th} jet

X = indicates vector cross product

$$\vec{MR} = \sum_{i=1}^{N} \vec{M}_i \tag{3}$$

$\vec{MR}$ = total effective moment from all RCS jets

N = number of RCS jets

$$\vec{F} = \sum_{i=1}^{N} \vec{T}_i \tag{4}$$

F = total effective linear force

$$A = R \sum_{j=1}^{3} \sum_{i=1}^{N} \left| T_i(j) \right|$$

A = reduction in mass of vehicle due to RCS fuel usage

R = RCS jet efficiency constant

2.14.4 Input/Output

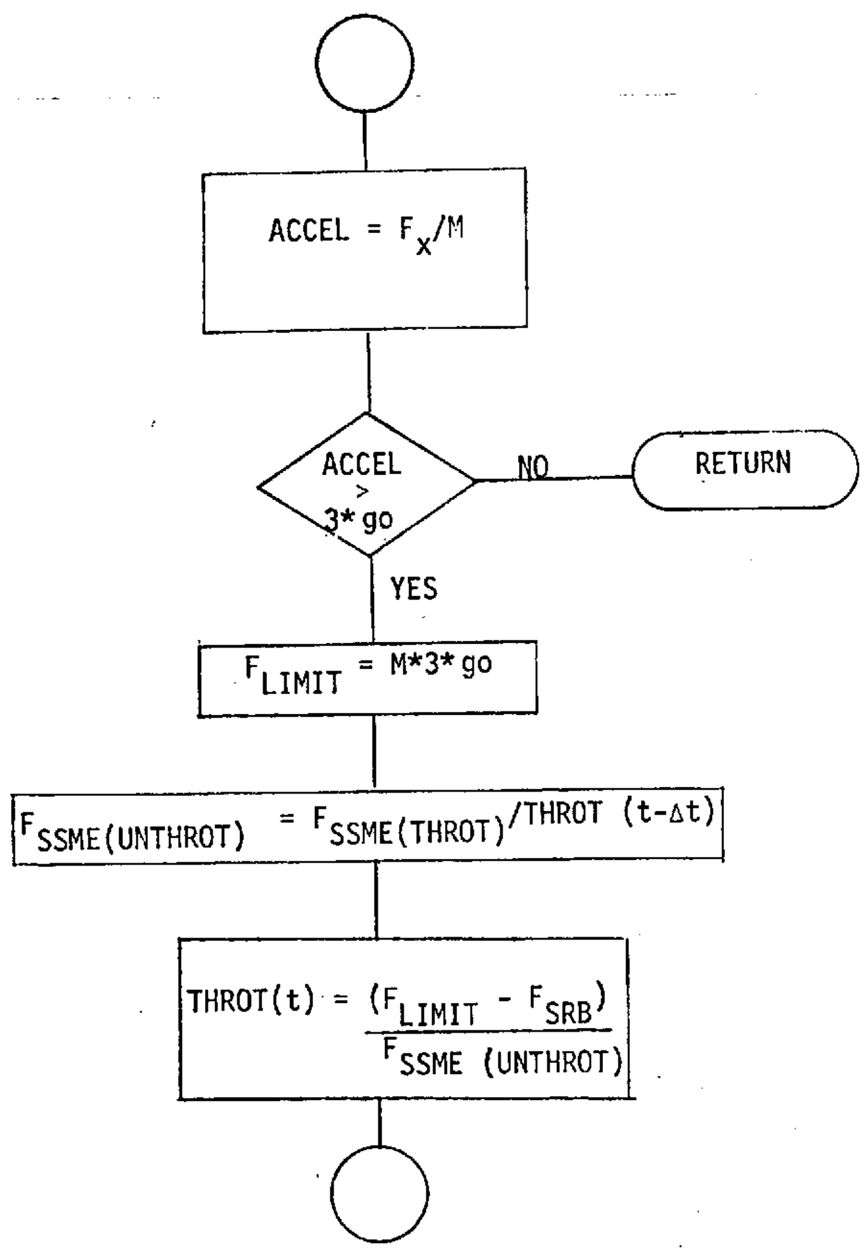
The model requires ELX, ELY, ELZ, XCG, YCG, ZCG, $\vec{T}_c$, t, and M as input. The flight software command accepted is $\vec{T}_c$. The cross product, mass properties, and EOM subroutines must be present. The model provides $\vec{MR}$, A, and $\vec{F}$ as output.

2.15 THRCMD (Throttle Command)

2.15.1 Program Description

This model senses vehicle longitudinal acceleration and issues throttle commands to the SSME's if the "3g" acceleration limit is exceeded. The throttle command is calculated such that the limit is maintained.

2.15.2 Math Model



2.15.3 Nomenclature

ACCEL	Vehicle acceleration along X body axis
F_x	Total thrust forces along X body axis
M	Vehicle mass
go	Accel. of gravity
F_{LIMIT}	Allowable thrust to maintain 3g acceleration
F_{SSME}	Sum of SSME X axis thrusts
F_{SRB}	Sum of SRB X Axis thrusts
THROT	Throttle command to SSME's

2.15.4 Input/Output

This model requires as input X axis thrusts for the SRB and SSME's. In addition the present throttle setting must be supplied. Outputs consist of identical throttle commands to the Space Shuttle main engines.

2.16 THRUST

2.16.1 Program Description

This model calculates forces and moments in body coordinates due to thrust forces from all engines given engine gimbal angles, throttle settings, and atmospheric pressure.

2.16.2 Math Model

Table lookup for SRM vacuum thrust, SRM mass flow rate, and power-on base drag.

$POBD = f\ (h)$

$time_{SRM} = time * (1 + UNBAL)$

$T_{V_{i\ (i\ =\ 4\text{-}5)}} = f\ (time_{SRM}) * (1 + UNBAL)$

$\dot{M}_{i\ (i\ =\ 4\text{-}5)} = f\ (time_{SRM}) * (1 + UNBAL)$

Calculate engine forces

$T_{i(i\ =\ 1\text{-}7)} = P_{T_i} * (T_{V_i} - EA_i * P_A)$

$\theta_P = \alpha_2 * (\tan \delta_{P_i} + \alpha_1 * (\tan \delta_{Y_i}))$

$\theta_Y = \alpha_2 * (\tan \delta_{Y_i} - \alpha_1 * (\tan \delta_{P_i}))$

if $i = 1\text{-}3,\ 6\text{-}7$ $\quad \alpha_1 = 0$ and $\alpha_2 = 1$

$i = 4,5$ $\quad \alpha_1 = 1$ and $\alpha_2 = .7071068$

(for $i = 4,5$; α_1 and α_2 are unique for diag. and SRB actuators)

$TBX_i = T_i * (1 + \theta_P{}^2 + \theta_Y{}^2)^{-1/2}$

$TBY_i = TBX_i * \theta_Y$

$TBZ_i = TBX_i * \theta_P$

Sum Forces

$$FB_X = \sum_{i=1,7} TBX_i + POBD$$

$$FB_Y = \sum_{i=1,7} TBY_i$$

$$FB_Z = \sum_{i=1,7} TBZ_i$$

Calculate moments

$$MTXB_i = TBZ_i\ (ELY_i - YCG) - TBY_i\ (ELZ_i - ZCG)$$

$$MTYB_i = TBX_i\ (ELZ_i - ZCG) - TBZ_i\ (ELX_i - XCG)$$

$$MTZB_i = TBY_i\ (ELX_i - XCG) - TBX_i\ (ELY_i - YCG)$$

Sum moments

$$MTB_X = \Sigma MTXB_i$$

$$MTB_Y = \Sigma MTYB_i$$

$$MTB_Z = \Sigma MTZB_i$$

2.16.3 Nomenclature

i = engine index; 1,2,3 = orbiter engines,
4,5 = solid engines
6,7 = OMS engines

POBD = Power-on base drag

h = Altitude

UNBAL = Individual SRM thrust unbalance

T_{V_i} = Vacuum thrust

$\dot{M}_i$ = Mass flow rate

T_i = Altitude thrust

P_{T_i} = Throttle setting

EA_i = Engine exit area

P_A = Atmospheric pressure

δ_{P_i} = Engine pitch gimbal angle

δ_{Y_i} = Engine yaw gimbal angle

TBX_i = Engine force in X body coordinate

TBY_i = Engine force in Y body coordinate

TBZ_i = Engine force in Z body coordinate

FB_X = Total thrust force in X body coordinate

FB_Y = Total thrust force in Z body coordinate

$MTXB_i$, $MTYB_i$, $MTZB_i$ = X, Y, and Z components, respectively, of moments due to engine number i.

ELX_i, ELY_i, ELZ_i = X, Y, and Z components, respectively, of engine locations in body coordinates.

XCG, YCG, ZCG = X, Y, and Z components of location of center of mass of the vehicle.

MTB_X, MTB_Y, and MTB_Z = X, Y, and Z components, respectively of total moments due to engine thrust

2.16.4 Input/Output

The parameters which must be supplied to the model as input are EA_i, ELX_i, ELY_i, ELZ_i, P_{Ti}, T_{Vi}, P_A, δ_{P_i}, δ_{Y_i}, XCG, YCG, and ZCG. The resulting

quantities calculated by the model are FTB and MTB. The flight software command accepted by the model is P_{T_i}. The outputs of this model are accepted by the equations of motion.

2.17 TSHAPE (Trajectory Shaping)

2.17.1 Program Description

The trajectory and control parameters which must be calculated to accomplish trajectory shaping are α_D, desired angle-of-attack; θ_C, desired pitch attitude angle; δ_D, desired engine deflection angle; and $\ddot{Z}_{DCG}$, desired body sensed acceleration. The values of θ_C, δ_D, and $\ddot{Z}_{DCG}$ are dependent on α_D. The boost flight is divided into three phases: vertical rise, tilt maneuver, and alpha policy. For each of these flight phases, α_D is calculated differently.

2.17.2 Math Model

2.17.3 General Calculations

$$C_{Z}\alpha_r = C_Z\alpha \cdot \frac{\pi}{180}$$

$$N_o = q\ S\ C_{ZO}$$

$$N'_P = q\ S\ C_Z\alpha_r$$

$$l_O = \bar{c}\ \ (C_{mo}/C_{zo}) + X_{CG} - X_R$$

$$l_{1P} = \bar{c}\ \ (C_m\alpha/C_z\alpha) + X_{CG} - X_R$$

$$D_Z = F_{ax}\ Z_{CG}$$

$$X_{1g} = X_{CG} - X_o$$

$$Z_{1g} = Z_{CG} - Z_o$$

$$L_o = (X_{1g}{}^2 + Z_{1g}{}^2)^{\frac{1}{2}}$$

$$L_B = \left[(X_B - X_{CG})^2 + (Z_B - Z_{CG})^2\right]^{1/2}$$

$$\delta_{CGB} = -\tan\left[\frac{Z_{CG} - Z_B}{X_{CG} - X_B}\right]$$

$$\delta_{CGO} = \tan\left[\frac{-Z_{1g}}{X_{1g}}\right]$$

Angle-of-Attack Calculations

Vertical Rise

$$\delta_D = \delta_{CGO} - L_B F_B \left(\frac{\delta_B - \delta_{CGB}}{L_0 F_0}\right)$$

$$\alpha_D = \tan^{-1}\left[\frac{F_0 \cos \delta_D = F_B \cos \delta_B}{-F_0 \sin \delta_D - F_B \sin \delta_B}\right]$$

$$\theta_C = \alpha_D$$

Tilt Maneuver (Parking Lot Tilt)

The maneuver modeled here is a modification of the tilt maneuver presented in the previous memo. Here, α_m is used as the value of angle-of-attack at a time half-way between the time to begin and end the tilt maneuver, T_T and T_D. Therefore, this procedure can fit any part of a parabola to the three points, depending on the value of α_m, and its relation to α_0 and α_D. This is illustrated in Figure 2-2.

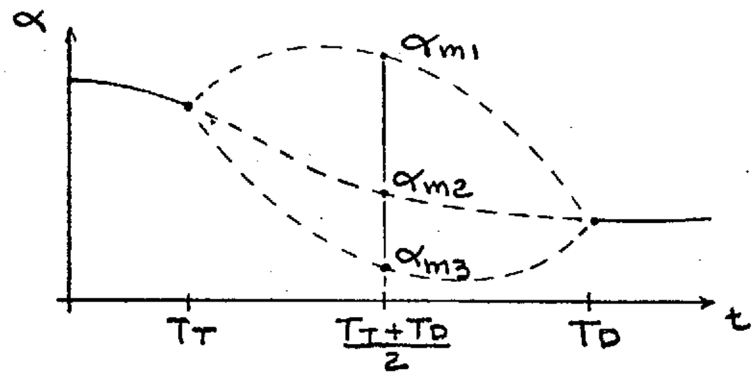


FIGURE 2-2

where α_{m1}, α_{m2}, and α_{m3} are three examples of values for α_m which cause a different shape curve to be fitted. The calculation for α_D during this maneuver is as follows:

$$\alpha_0 = \tan\left[\frac{V_{BZ}}{V_{BX}}\right], \text{ calculated only at } t = T_T$$

$$T_{OP} = \frac{T_T + T_D}{2}$$

$$A_1 = \frac{\alpha_D - \alpha_0}{2}$$

$$A_2 = \frac{(\alpha_D - \alpha_m) + (\alpha_D - \alpha_m)}{2}$$

$$Z = \frac{T - T_{OP}}{\frac{1}{2}\,(T_D - T_T)}$$

$$\alpha_D = \alpha_m + A_1 Z + A_2 Z^2$$

$$K_2 = \frac{-N'_P + F_T}{m}$$

$$K_3 = \frac{F_0}{m}$$

$$\ddot{\theta}_o = \frac{l_o N_o - F_{ax} (Z_{CG} - Z_R)}{I_{yy}}$$

$$\ddot{Z}_o = N_o/m$$

$$B_1 = C_{2P} \delta_{CGO} + \ddot{\theta} - C_{2B}$$

$$B_2 = \ddot{Z}_o - K_{3B}$$

$$\alpha_D = \frac{B_1 K_3 - B_2 C_{2P}}{C_{1P} K_3 - C_{2P} K_2}$$

Aerodynamic Moment Control

$$\alpha_D = \frac{-l_o N_o + D_z}{l_{1P} N'_P}$$

Remaining Shaping and Control Parameters

The following parameters are calculated after a value for α_D has been determined by some specified alpha policy.

$$\theta_R = \tan^{-1}\left[\frac{V_{RZ}}{V_{RX}}\right]$$

$$\theta c = \alpha_D - \theta_R$$

$$\ddot{\theta}_o = \frac{l_o N_o - F_{ax}\,(Z_{CG} - Z_R)}{I_{yy}}$$

$$C_{1P} = \frac{-l_{1P}\, N'_P}{I_{yy}}$$

$$\delta_D = \frac{(\ddot{\theta}_o - C_{1P}\,\alpha_D)\; I_{yy}}{L_o F_o} + \delta_{CGO} \;\frac{- L_B F_B (\delta_B - \delta_{CGB})}{L_o F_o}$$

$$\ddot{Z}_{DCG} = \frac{1}{m}\left[N_o + N'_P\;\alpha D - F_o \sin\delta_D - F_B \sin\delta_B\right]$$

2.17.3 Nomenclature

Variable	Definition
A_1, A_2	Temp. variables
B_1, B_2	Temp. variables
C_{1P}, C_{2P}, C_{2B}	Temp. variables
$\bar{c}$	Mean aerodynanic cord
C_{mo}, C_m	Aero. moment coefficients
C_{ZO}, $C_{Z\alpha}$	Aero. normal force coefficients (pitch)

Variable	Definition
D_Z	Drag moment due to CG offset
F_T	Total force acting on vehicle
F_{ax}	Aeoo. axial force
I	Moments of inertia
K_2, K_3, K_{3B}	Temp. variables
L_O, L_B	Moment arms from CG to orbiter and booster engines
m	Mass
N_o	Normal force at zero angle-of-attack
N'_P	Partial of normal
q	Dynamic pressure
S	Aerodynamic reference area
T	Present time
T_D	Time to end tilt maneuver (begin alpha policy)
T_{OP}	Mid point between T_D and T_T
T_T	Time to begin tilt maneuver (end vertical rise)
V_{BX}, V_{BZ}	Velocity components in body coordinates
V_{RX}, V_{RZ}	Inertial components of velocity relative to air
X_{CG}, Y_{CG}, Z_{CG}	Location of center of gravity
X_{1g}, Z_{1g}	Moment arm from CG to orbiter engines
X_o, Z_o	Average orbiter engine location
X_R, Z_R	Location of aerodynamic reference
X_a	Location of body fixed accelerometer
Z	Temp. variable
$\ddot{Z}_{DCG}$	Desired body sensed acceleration

Variable	Definition
α_D	Desired angle-of-attack
α_m	Value of angle-of-attack at T_{OP}
α_o	Angle-of-attack at T_T
δ_{CGB}, δ_{CGO}	Deflection for thrust through center of gravity at booster and orbiter engines
δ_D	Desired orbiter engine deflection angle
θ_c	Commanded pitch angle
$\ddot{\theta}_o$	Acceleration at zero angle-of-attack

2.17.4 Input/Output

The inputs necessary for TSHAPE are the aerodynamic constants, configuration constants, I, m, CG's, T, T_D, T_T, V_B, V_R, and α_m. The outputs of this program are α_D, θ_C, δ_D, $\ddot{Z}_{DG}$, δ_{CGB}, and δ_{CGO}.

2.18 TVC (Thrust Vector Control)

2.18.1 Program Description

This model describes the motions of massless engines with limits on deflection, deflection rate, and acceleration.

2.18.2 Math Model

Initialization (time = 0)

$$\delta P_{OLD_i\ (i=1,7)} = \delta P_{NEW_i}$$

$$\delta Y_{OLD_i\ (i=1,7)} = \delta Y_{NEW_i}$$

Process and Limit Commands

if $(\delta_{P_{COMMAND_i}} - PNULL_i) > PLIMIT_i$

$$\delta_{P_{COMMAND_i}} = PLIMIT_i + PNULL_i$$

if $(\delta_{Y_{COMMAND_i}} - YNULL_i) > YLIMIT_i$

$$\delta_{Y_{COMMAND_i}} = YLIMIT + YNULL_i$$

$$\delta_{P_{NEW_i}} = (1-e^{-10*\Delta time}) * \delta_{P_{COMMAND_i}} + (e^{-10*\Delta time}) * \delta_{P_{OLD_i}}$$

$$\delta_{Y_{NEW_i}} = (1-e^{-10*\Delta time}) * \delta_{Y_{COMMAND_i}} + (e^{-10*\Delta time}) * \delta_{Y_{OLD_i}}$$

Calculate and Limit Rates

$$\dot{\delta}_{P_{NEW_i}} = (\delta_{P_{NEW_i}} - \delta_{P_{OLD_i}})/\Delta time$$

limit to $\dot{\delta}_{P_{LIMIT_i}}$

$$\dot{\delta}_{Y_{NEW_i}} = (\delta_{Y_{NEW_i}} - \delta_{Y_{OLD_i}})/\Delta time$$

limit to $\dot{\delta}_{Y_{LIMIT_i}}$

Calculate and Limit Accelerations

$$\ddot{\delta}_{P_i} = (\dot{\delta}_{P_{NEW_i}} - \dot{\delta}_{P_{OLD_i}})/\Delta time$$

limit to $\ddot{\delta}_{P_{LIMIT_i}}$

$$\ddot{\delta}_{Y_i} = (\dot{\delta}_{Y_{NEW_i}} - \dot{\delta}_{Y_{OLD_i}})/\Delta time$$

limit to $\ddot{\delta}_{Y_{LIMIT_i}}$

Define new rate using limited acceleration

$$\dot{\delta}_{P_{NEW_i}} = \dot{\delta}_{P_{OLD_i}} + (\ddot{\delta}_{P_{NEW_i}}) * (\Delta time)$$

$$\dot{\delta}_{Y_{NEW_i}} = \dot{\delta}_{Y_{OLD_i}} + (\ddot{\delta}_{Y_{NEW_i}}) * (\Delta time)$$

Define new position using limited rate

$$\delta_{P_{NEW_i}} = \delta_{P_{OLD_i}} + (\dot{\delta}_{P_{NEW_i}}) * (\Delta time) + BIASP_i$$

$$\delta_{Y_{NEW_i}} = \delta_{Y_{OLD_i}} + (\dot{\delta}_{Y_{NEW_i}}) * (\Delta time) + BIASY_i$$

Reset Old Values

$$\delta_{P_{OLD_i}} = \delta_{P_{NEW_i}} - BIASP_i$$

$$\delta_{Y_{OLD_i}} = \delta_{Y_{OLD_i}} - BIASY_i$$

$$\dot{\delta}_{P_{OLD_i}} = \dot{\delta}_{P_{NEW_i}}$$

$$\dot{\delta}_{Y_{OLD_i}} = \dot{\delta}_{Y_{NEW_i}}$$

Calculate Duty Cycle

$$DCYCLE_i = DCYCLE_i + (\Delta time) * (|\dot{\delta}_{P_{NEW_i}}| + |\dot{\delta}_{Y_{NEW_i}}|)$$

2.18.3 Nomenclature

$\delta_{P_{NEW}}$ = present pitch deflection

$\delta_{Y_{NEW}}$ = present yaw deflection

$\delta_{P_{OLD}}$ = past pitch deflection

$\delta_{Y_{OLD}}$ = past yaw deflection

$\dot{\delta}_{P_{NEW}}$ = present pitch deflection rate

$\dot{\delta}_{Y_{NEW}}$ = present yaw deflection rate

$\dot{\delta}_{P_{OLD}}$ = past pitch deflection rate

$\dot{\delta}_{Y_{OLD}}$ = past yaw deflection rate

$\ddot{\delta}_{P}$ = pitch deflection acceleration

$\ddot{\delta}_{Y}$ = yaw deflection acceleration

BIASP = engine pitch bias due to misalignments

BIASY = engine yaw bias due to misalignments

PLIMIT = pitch deflection limit

YLIMIT = yaw deflection limit

PNULL = pitch null position

YNULL = yaw null position

$\delta_{P_{COMMAND}}$ = pitch deflection command from flight computer

$\delta_{Y_{COMMAND}}$ = yaw deflection command from flight computer

$\dot{\delta}_{P_{LIMIT}}$ = pitch rate limit

$\dot{\delta}_{Y_{LIMIT}}$ = yaw rate limit

$\ddot{\delta}_{P_{LIMIT}}$ = pitch acceleration limit

$\ddot{\delta}_{Y_{LIMIT}}$ = yaw acceleration limit

i = engine index; 1, 2, 3 = orbiter engines

4, 5 = solid engines

6, 7 = OMS engines

2.18.4 Input/Output

Inputs: Pitch and yaw deflection commands from the flight control system (for each engine); deflection, rate, and acceleration limits; pitch and yaw null positions; and canned yaw bias.

Outputs: Engine deflections, deflection rates, deflection accelerations, and duty cycle requirements.

<-2

3.0 COORDINATE SYSTEMS

Inertial Polar-Equatorial - A right-handed orthogonal system with its origin at the center of the earth - X axis in the equatorial plane and positive through a reference meridian at the time of liftoff; the reference meridian is defined by the time of liftoff and the coordinate system used for gravity calculations. The Z axis is positive through the North Pole.

Inertial Plumbline - An orthogonal system with its origin at the center of the earth, X axis parallel to the launch site gravity vector and positive in the direction opposite to gravitational acceleration. The Z axis lies in the launch plane and points downrange and the Y axis completes a right-handed triad.

Local Vertical - An orthogonal system with its origin at the center of the earth, the X axis points from the earth center to the vehicle, the Z axis is in the plane containing the earth's rotation axis and the X_{LV} axis. The Z axis is perpendicular to the X axis and points towards the North Pole. The Y axis completes a right-handed triad.

Body - An orthogonal system with its origin at the engine gimbal pivot plane - X axis positive towards the nose of the vehicle along the main propellant tank centerline, Z axis positive "down", and the Y axis completes the right-handed system and is positive in the direction of the right wing.

Transformation matrix from polar-equatorial to plumbline coordinates:

$$\left[\alpha\right] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{11} = \cos \lambda_L^* \cos (\phi_L^* + \omega_e t_L)$$

$$a_{12} = \cos \lambda_L^* \sin (\phi_L^* + \omega_e t_L)$$

$$a_{13} = \sin \lambda_L^*$$

$$a_{21} = \sin A_L \sin \lambda_L^* \cos (\omega_e t_L + \phi_L^*) - \cos A_L \sin (\omega_e t_L + \phi_L^*)$$

$$a_{22} = \sin A_L \sin \lambda_L^* \sin (\omega_e t_L + \phi_L^*) - \cos A_L \cos (\omega_e t_L + \phi_L^*)$$

$$a_{23} = -\sin A_L \cos \lambda_L^*$$

$$a_{31} = -\cos A_L \sin \lambda_L^* \cos (\omega_e t_L + \phi_L^*) - \sin A_L \sin (\omega_e t_L + \phi_L^*)$$

$$a_{32} = -\cos A_L \sin \lambda_L^* \sin (\omega_e t_L + \phi_L^*) + \sin A_L \cos (\omega_e t_L + \phi_L^*)$$

$$a_{33} = \cos A_L \cos \lambda_L^*$$

Where:

λ_L^* = geodetic latitude of launch site

ϕ_L^* = longitude of launch site

ω_e = angular rate of earth

T_L = time of launch (from epoch)

A_L = launch azimuth

Transformation matrix from body to inertial plumbline coordinates:

$$\left[\beta\right] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$b_{11} = \cos\theta \cos\psi$

$b_{12} = \sin\theta \sin\phi - \cos\theta \sin\psi \cos\phi$

$b_{13} = \sin\theta \cos\phi + \cos\theta \sin\psi \sin\phi$

$b_{21} = \sin\psi$

$b_{22} = \cos\psi \cos\phi$

$b_{23} = -\cos\psi \sin\phi$

$b_{31} = -\sin\theta \cos\psi$

$b_{32} = \cos\theta \sin\phi + \sin\theta \sin\psi \cos\phi$

$b_{33} = \cos\theta \cos\phi - \sin\theta \sin\psi \sin\phi$

where the Euler angles θ, ψ and ϕ are calculated in EØM.

Transformation matrix from local vertical to polar-equatorial coordinates:

$$\left[\phi\right] = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

$$d_{11} = \cos\lambda_V \cos\phi'$$

$$d_{12} = -\sin\phi'$$

$$d_{13} = -\sin\lambda_V \cos\phi'$$

$$d_{21} = \cos\lambda_V \sin\phi'$$

$$d_{22} = \cos\phi'$$

$$d_{23} = -\sin\lambda_V \sin\phi'$$

$$d_{31} = \sin\lambda_V$$

$$d_{32} = 0$$

$$d_{33} = \cos\lambda_V$$

Where:

$$\sin\lambda_V = Z_F/R$$

$$\cos\lambda_V = \sqrt{1 - \sin^2\lambda_V}$$

$$\sin\phi' = Y_F/R\cos\lambda_V$$

$$\cos\phi' = X_F/R\cos\lambda_V$$

4.0 FLEXIBLE BODY MATH MODELS

Mathematical models were developed for use in the SSFS. These models were programmed for a parallel burn solid rocket motor configuration. The flexible body version of SSFS is currently being checked out.

4.1 Flexible Body Program Description

This program contains the bending and slosh models for the launch configuration during first stage boost. It uses a generalized modal approach to bending which represents the elastic response by standard normal modal equations with viscous damping. Included are models for aerodynamic forces and moment and thrust forces and moments to account for bending effects as well as the tail wags dog contribution to bending. The rigid body and elastic response equations provided here are uncoupled and are considered separately since the magnitude of the coupling is insignificant. The number of equations is very sensitive to the vehicle configuration and to the completeness of the bending analysis. Therefore, when data becomes available it is likely that only a small percentage of the general set provided here will actually be required for SSV analysis.

The model sums all the forces acting on each of the equivalent mass points and for a given mode numerically integrates the sum with a second order linear differential equation in modal displacement. The number of mass points at which aero forces and modal displacements are calculated will be less than 50. The number of modes at these points will be less than 10 each. The number of slosh masses will be less than 5 and the number of modes per slosh mass will be less than 5. The EOM, guidance atmosphere and control subroutine must be present to provide inputs for this model.

4.2 Vibration Equations

$$\sum_{j=1}^{N1} (F_{axj}\ \phi_{xij} + F_{ayj}\ \phi_{yij} + F_{azj}\ \phi_{zij})$$

$$+ \sum_{j=1}^{M1} (F_{txj}\ \phi_{xij} + F_{tyj}\ \phi_{yij} + F_{tzj}\ \phi_{zij})$$

$$+ \sum_{j=1}^{K1} (F_{sxj}\ \phi_{xij} + F_{syj}\ \phi_{yij} + F_{szj}\ \phi_{zij})$$

$$+ \sum_{j=1}^{N2} (M_{axj}\ \phi'_{xij} + M_{ayj}\ \phi'_{yij} + M_{azj}\ \phi'_{zij})$$

$$+ \sum_{j=1}^{M2} (M_{txj}\ \phi'_{xij} + M_{tyj}\ \phi'_{yij} + M_{tzj}\ \phi'_{zij})$$

$$+ \sum_{j=1}^{K2} (M_{sxj}\ \phi'_{xij} + M_{syj}\ \phi'_{yij} + M_{szj}\ \phi'_{zij})$$

$$= m_i\ (\ddot{q}_i + 2\ \zeta_i\ \omega_i\ \dot{q}_i + \omega_i^2\ q_i)$$

$$\dot{q}_i = \int \ddot{q}_i\ dt + \dot{q}_i$$

$$q_i = \int \dot{q}_i\ dt + q_i$$

Where:

$N1$ = number of aero stations for aero forces

$N2$ = number of aero stations for aero moments

$M1$ = number of engines producing forces

$M2$ = number of engines producing moments

$K1$ = number of slosh stations for slosh forces

$K2$ = number of slosh stations for slosh moments

q_i = modal displacement due to bending mode i

ζ_i = damping coefficient for mode i

ω_i = frequency of mode i

m_i = normalized mass for mode i

F_{axj} = aero forces in X direction at station j

F_{ayj} = aero forces in Y direction at station j

F_{azj} = aero forces in Z direction at station J

F_{txj} = thrust forces in X direction for engine j

F_{tyj} = thrust forces in Y direction for engine j

F_{tzj} = thrust forces in Z direction for engine j

F_{sxj} = slosh forces in X direction at station j

F_{syj} = slosh forces in Y direction at station j

F_{szj} = slosh forces in Z direction at station j

M_{axj} = aero moments about X axis at station j

M_{ayj} = aero moments about y axis at station j

M_{azj} = aero moments about z axis at station j

M_{txj} = thrust moments about X axis due to engine j

M_{tyj} = thrust moments about Y axis due to engine j

M_{tzj} = thrust moments about Z axis due to engine j

M_{sxj} = slosh moments about X axis at station j

M_{syj} = slosh moments about Y axis at station j

M_{szj} = slosh moments about Z axis at station j

ϕ_{xij} = mode shape translation in X direction for mode i at location j

ϕ_{yij} = mode shape translation in Y direction for mode i at location j

ϕ_{zij} = mode shape translation in Z direction for mode i at location j

ϕ'_{xij} = mode slope about X axis for mode i at location j

ϕ'_{yij} = mode slope about Y axis for mode i at location j

ϕ'_{zij} = mode slope about Z axis for mode i at location j

4.3 Aerodynamic Forces

$$\begin{bmatrix} V_{wxpj} \\ V_{wypj} \\ V_{wzpj} \end{bmatrix} = [\alpha] \quad [\delta] \quad \begin{bmatrix} 0 \\ -V_{w_j} \sin A_{zw_j} \\ -V_{w_j} \cos A_{zw_j} \end{bmatrix}$$

$$\begin{bmatrix} V'_{axbj} \\ V'_{aybj} \\ V'_{azbj} \end{bmatrix} = [\beta]^{-1} \left[\begin{bmatrix} V_{R_{X_P}} \\ V_{R_{X_P}} \\ V_{R_{X_P}} \end{bmatrix} - \begin{bmatrix} V_{wxpj} \\ V_{wypj} \\ V_{wzpj} \end{bmatrix} \right]$$

$$V_{axbj}'' = Q\,(\bar{Z}_j - \bar{Z}_{cg}) - R\,(\bar{Y}_j - \bar{Y}_{cg})$$

$$V_{aybj}'' = R\,(\bar{X}_j - \bar{X}_{cg}) - P\,(\bar{Z}_j - \bar{Z}_{cg})$$

$$V_{azbj}'' = P\,(\bar{Y}_j = \bar{Y}_{cg}) - Q\,(\bar{X}_j - \bar{X}_{cg})$$

$$V_{axbj}''' = \sum_{i=1}^{M3} \phi_{xij}\,\dot{q}_i$$

$$V_{aybj}''' = \sum_{i=1}^{M3} \phi_{yij}\,\dot{q}_i$$

$$V_{azbj}''' = \sum_{i=1}^{M3} \phi_{zij}\,\dot{q}_i$$

$$V_{axbj}^{IV} = V_{aybj} \sum_{i=1}^{M3} \phi'_{zij}\,q_i - V_{azbj} \sum_{i=1}^{M3} \phi'_{yij}\,q_i$$

$$V_{aybj}^{IV} = V_{azbj} \sum_{i=1}^{M3} \phi'_{xij}\,q_i - V_{axbj} \sum_{i=1}^{M3} \phi'_{zij}\,q_i$$

$$V_{azbj}^{IV} = V_{axbj} \sum_{i=1}^{M3} \phi'_{yij}\,q_i - V_{aybj} \sum_{i=1}^{M3} \phi'_{xij}\,q_i$$

$$V_{axbj} = V_{axbj}' + V_{axbj}'' + V_{axbj}''' + V_{axbj}^{IV}$$

$$V_{aybj} = V_{aybj}' + V_{aybj}'' + V_{aybj}''' + V_{aybj}^{IV}$$

$$V_{azbj} = V_{azbj}' + V_{azbj}'' + V_{azbj}''' + V_{azbj}^{IV}$$

The previous 6 equations must be solved simultaneously for V_{axbj}, V_{aybj}, and V_{azbj}. The solution is as follows:

let

$$a_{11} = -1$$

$$a_{12} = \sum_{i=1}^{M3} \phi_{zij}' q_i$$

$$a_{13} = -\sum_{i=1}^{M3} \phi_{yij}' q_i$$

$$a_{21} = -\sum_{i=1}^{M3} \phi_{zij}' q_i$$

$$a_{22} = -1$$

$$A_{23} = +\sum_{i=1}^{M3} \phi_{xij}' q_i$$

$$a_{31} = \sum_{i=1}^{M3} \phi'_{yij}\, q_i$$

$$a_{32} = -\sum_{i=1}^{M3} \phi'_{xij}\, q_i$$

$$a_{33} = -1$$

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then:

$$\begin{bmatrix} V_{axbj} \\ V_{aybj} \\ V_{azbj} \end{bmatrix} = [A]^{-1} \begin{bmatrix} -V_{axbj}^{'} - V_{axbj}^{''} - V_{axbj}^{'''} \\ -V_{aybj}^{'} - V_{aybj}^{''} - V_{aybj}^{'''} \\ -V_{azbj}^{'} - V_{azbj}^{''} - V_{azbj}^{'''} \end{bmatrix}$$

$$V_{aj} = \sqrt{V_{axbj}^{2} + V_{aybj}^{2} + V_{azbj}^{2}}$$

$$\beta_j = \text{Arcsin}\left(\frac{V_{aybj}}{V_{aj}}\right)$$

$$\alpha_j = \text{Arctan}\left(\frac{V_{azbj}}{V_{axbj}}\right)$$

$$M_j = \frac{V_{aj}}{a}$$

$$q_j = \tfrac{1}{2} \rho V_{aj}^2$$

$$F_{axj} = q_j S (C_{xoj} + C_{x\alpha j} \alpha_j)$$

$$F_{ayj} = q_j S C_{y\beta j} \beta_j$$

$$F_{azj} = q_j S (C_{zoj} + C_{z\alpha j} \alpha_j)$$

Where:

V_{w_j} = magnitude of relative wind at Station j

A_{zw_j} = azimuth angle of relative wind at Station j

V_{wxpj}, V_{wypj}, V_{wzpj} = relative wind velocity at mass point j in platform coordinates

$[\alpha]$ = transformation described in "coordinate systems"

$[\beta]$ = transformation described in "coordinate systems"

$[\delta]$ = transformation described in "coordinate systems"

V_{axbj}',

V_{aybj}', = components of air velocity in body coordinates at mass point j

V_{azbj}'

P, Q, R = angular velocity about X, Y, and Z axes respectively

$\bar{X}_j$, $\bar{Y}_j$, $\bar{Z}_j$ = location of mass point j in body coordinates

$\bar{X}_{cg}$, $\bar{Y}_{cg}$, $\bar{Z}_{cg}$ = location of center of gravity in body coordinates

V_{axbj}'',

V_{aybj}'', = component of velocity at mass point j due to rotation of mass point about c.g.

V_{azbj}''

V_{axbj}''',

V_{aybj}''', = components of velocity of vibrating mass with respect to rigid body

V_{azbj}'''

V_{axbj}^{IV},

V_{aybj}^{IV}, = components of velocity due to perpendicular forces being rotated with respect to rigid body due to bending

V_{azbj}^{IV}

M3 = number of bending modes

$V_{R_{X_P}}$, $V_{R_{Y_P}}$, $V_{R_{Z_P}}$ = velocity of vehicle relative to the earth in platform coordinate

V_{axbj},

V_{aybj}, = components of velocity of mass point j with respect to air

V_{azbj}

α_j = angle of attack of mass point j

M_j = mach number of mass point j

V_{aj} = velocity of mass point j with respect to air

a = speed of sound

q_j = dynamic pressure at mass point j

ρ = mass density of air

S = aero reference area

$$\left.\begin{array}{l} C_{xoj}, \\ C_{x\alpha j}, \\ C_{y\beta j}, \\ C_{zoj}, \\ C_{z\alpha j}, \end{array}\right\} = \text{aero coefficients for mass point j}$$

β_j = sideslip angle for mass point j

4.4 Engine Forces

$$\theta_{yj} = \theta'_{yj} + \sum_{i=1}^{M3} \phi'_{zij} q_i$$

$$\theta_{pj} = \theta'_{pj} + \sum_{i=1}^{M3} \phi'_{yij} q_i$$

$$T_{bxj} = ET_j \cos \theta_{pj} \cos \theta_{yj}$$

$$T_{byj} = ET_j \sin \theta_{yj}$$

$$T_{bzj} = -ET_j \sin \theta_{pj} \cos \theta_{yj}$$

Limit θ_{pcj} and θ_{ycj} to position limit

$$\dot{\hat{\theta}}_{pj} = \left[\omega_a ;(\theta_{pcj} - \hat{\theta}_{pj}) + K_{2j}(\theta'_{pj} - \hat{\theta}_{pj}) + K_{1j} \dot{\theta}'_{pj}\right] /(1+K_{1j})$$

$$\dot{\hat{\theta}}_{yj} = \left[\omega_a ;(\theta_{ycj} - \hat{\theta}_{yj}) + K_{2j}(\theta'_{yj} - \hat{\theta}_{yj}) + K_{1j} \dot{\theta}'_{yj}\right] /(1+K_{1j})$$

Limit $\dot{\hat{\theta}}$ to rate limit

If $\hat{\theta}$ + $\dot{\hat{\theta}}$dt exceed position limit, limit $\dot{\hat{\theta}}$ to (position limit $-\hat{\theta}$)/dt

$$\hat{\theta}_{pj} = \int \dot{\hat{\theta}}_{pj}\, dt + \hat{\theta}_{pjo}$$

$$\hat{\theta}_{yj} = \int \dot{\hat{\theta}}_{yj}\, dt + \hat{\theta}_{yjo}$$

$$\dot{\theta}'_{pj} + \dot{Q} + \sum_{i=1}^{M3} \phi'_{yij} \ddot{q}_i + 2\zeta_{ep} \omega_{ep} \dot{\theta}'_{pj} + \omega^2_{ep} \theta_{pj} =$$

$$\omega^2_{ep} \hat{\theta}_{pj} - \frac{F_{ezj}}{I_{yye}} (X_{ecgj} - X_{ej})$$

$$\ddot{\theta}'_{yj} + \dot{R} + \sum_{i=1}^{M3} \phi'_{zij} \ddot{q}_i + 2 \zeta_{ej} \omega_{ey} \theta'_{yj} + \omega^2_{ey} \theta'_{yj} =$$

$$\omega^2_{ey} \hat{\theta}_{yj} + \frac{F_{eyj}}{I_{zze}} (X_{ecgj} - X_{ej})$$

$$\dot{\theta}'_{pj} = \int \ddot{\theta}'_{pj} \, dt + \dot{\theta}'_{pjo}$$

$$\theta'_{pj} = \int \dot{\theta}'_{pj} \, dt + \theta'_{pjo}$$

$$\dot{\theta}'_{yj} = \int \dot{\theta}'_{yj} \, dt + \theta'_{yjo}$$

$$\dot{\theta}'_{yj} = \int \ddot{\theta}'_{yj} \, dt + \dot{\theta}_{yjo}$$

$$F'_{txj} = ET_j \sqrt{1 + TAN^2 \theta'_p + TAN^2 \theta'_y}$$

$$F'_{tyj} = E_{tj} \, TAN \, \theta'_y$$

$$F'_{tzj} = -E_{tj} \, TAN \theta'_p$$

$$F_{exj} = -m_{ej} \quad A_x + \sum_{i=1}^{M3} \phi_{xij} \ddot{q}_i - (y_{ej} - y_{cg}) \dot{R} + (Z_{cj} - Z_{cg}) \dot{Q}$$

$$F_{eyj} = -m_{ej} \left\{ A_y + (X_{ej} - X_{CG}) \dot{R} - (Z_{ej} - Z_{CG}) \dot{P} + \sum_{i=1}^{M3} \phi_{yij} \ddot{q}_i \right\}$$

$$F_{ezj} = -m_{ej} \left\{ A_z - (X_{ej} - X_{cg}) \dot{Q} + (y_{ej} - y_{cg}) \dot{P} + \sum_{i=1}^{M3} \phi_{zij} \ddot{q}_i \right\}$$

$$F_{txj} = F'_{txj} + F_{exj}$$

$$F_{tyj} = F'_{tyj} + m_{ej} (X_{ej} - X_{ecg}) \ddot{\theta}'_y + F_{eyj}$$

$$F_{tzj} = F'_{tzj} + m_{ej} (X_{ecg} - X_{ej}) \ddot{\theta}'_p + F_{ezj}$$

Where:

θ'_{yj} = yaw engine gimbal angle with respect to the mounting surface for engine j

θ_{yj} = yaw engine gimbal angle with respect to rigid body coordinates for engine j

θ'_{pj} = pitch engine gimbal angle with respect to the mounting surface for engine j

θ_{pj} = pitch engine gimbal angle with respect to rigid body coordinates for engine j

$\left.\begin{matrix} \bar{X}_{ej}, \\ \bar{Y}_{ej}, \\ \bar{Z}_{ej} \end{matrix}\right\}$ = location of engine j pivot point

$\left.\begin{array}{l}\bar{X}_{ecgj},\\ \bar{Y}_{ecgj},\\ \bar{Z}_{ecgj}\end{array}\right\}$ = location of engine j center of mass

$\left.\begin{array}{l}T_{bxj},\\ T_{byj},\\ T_{bzj}\end{array}\right\}$ = thrust forces acting on vehicle

$\hat{\theta}_{pj}$ = pitch engine actuator angle

$\hat{\theta}_{yj}$ = yaw engine actuator angle

$\hat{\dot{\theta}}_{pj}$ = pitch engine actuator rate

I_{yye} = moment of inertia of engine bell about Y axis at engine gimbal point

I_{zze} = moment of inertia of engine bell about Z axis at engine gimbal point

$\hat{\dot{\theta}}_{yj}$ = yaw engine actuator rate

θ_{pcj} = pitch gimbal angle command

θ_{ycj} = yaw gimbal angle command

ζ_{ep} = damping factor for pitch engine dynamics

ζ_{ey} = damping factor for yaw engine dynamics

ω_e = frequency for engine dynamics

ω_a = frequency for actuator dynamics

$\dot{\theta}_p$ = present engine rate for pitch

$\dot{\theta}_y$ = present engine rate for yaw

$\ddot{\theta}_p$ = pitch engine angular acceleration

$\ddot{\theta}_y$ = yaw engine angular acceleration

M_{ej} = mass of engine j

A_x, A_y, A_z = linear acceleration of vehicle in body coordinates

$\left.\begin{array}{l}\bar{X}_{cg},\\ \bar{Y}_{cg},\\ \bar{Z}_{cg}\end{array}\right\}$ = vehicle center of gravity

The characteristicsfrequencies associated with the engine dynamics are much higher than the vehicle characteristic frequencies. It is recommended that the engine dynamics be integrated separately with an integration cycle of 50 milliseconds.

4.5 Slosh Forces

$$\ddot{\lambda}_{xj} + 2\zeta_{sj}\,\omega_{sj}\,\dot{\lambda}_{xj} + \omega^2_{sj}\,\lambda_{xj} = -\sum_{i=1}^{M3} \phi_{xij}\,\ddot{q}_i - A_x - \dot{Q}\,(Z_{sj} - \bar{Z}_{cg})$$

$$+ \dot{R}\,(Y_{sj} - \bar{Y}_{cg})$$

$$\ddot{\lambda}_{yj} + 2\zeta_{sj}\,\omega_{sj}\,\dot{\lambda}_{yj} + \omega^2_{sj}\,\lambda_{yj} = -\sum_{i=1}^{M3} \phi_{yij}\,\ddot{q}_i - A_y - \dot{R}\,(X_{sj} - \bar{X}_{cg})$$

$$+ \dot{P}\,(Z_{sj} - \bar{Z}_{cg})$$

$$\ddot{\lambda}_{zj} + 2\,\zeta_{sj}\,\omega_{sj}\,\dot{\lambda}_{zj} + \omega^2_{sj}\,\lambda_{zj} = -\sum_{i=1}^{M3} \phi_{zij}\,\ddot{q}_i - A_Z - \dot{P}\,(Y_{sj} - \overline{Y}_{cg})$$

$$+ \dot{Q}\,(X_{sj} - \overline{X}_{cg})$$

$$F_{sxj} = -m_{sj}\,\ddot{\lambda}_{xj}$$

$$F_{xyj} = -m_{sj}\,\ddot{\lambda}_{yj}$$

$$F_{szj} = -m_{sj}\,\ddot{\lambda}_{zj}$$

$$\dot{\lambda}_{xj} = \int \ddot{\lambda}_{xj}$$

$$\dot{\lambda}_{yj} = \int \ddot{\lambda}_{yj}$$

$$\dot{\lambda}_{zj} = \int \ddot{\lambda}_{zj}$$

$$\lambda_{xj} = \int \dot{\lambda}_{xj}$$

$$\lambda_{yi} = \int \dot{\lambda}_{yj}$$

$$\lambda_{zj} = \int \dot{\lambda}_{zj}$$

Where:

$\lambda_{xj}, \lambda_{yj}, \lambda_{zj}$ = displacements of slosh mass j

$\dot{\lambda}_{xj}, \dot{\lambda}_{yj}, \dot{\lambda}_{zj}$ = velocities of slosh mass j

$\ddot{\lambda}_{xj}, \ddot{\lambda}_{yj}, \ddot{\lambda}_{zj}$ = acceleration of slosh mass j

ζ_{sj} = damping factor for slosh mode j

ω_{sj} = characteristic frequency of slosh mode j

X_{sj},
Y_{sj}, = position of slosh mass j
Z_{sj}

m_{sj} = mass of sloshing fluid at mode j

4.6 Aerodynamic Moments

$$M'_{axj} = F_{ayj} (Z_{cg} - Z_{arj}) - F_{azj} (Y_{cg} - Y_{arj})$$

$$M'_{ayj} = F_{azj} (X_{cg} - X_{arj}) - F_{axj} (Z_{cg} - Z_{arj})$$

$$M'_{azj} = F_{axj} (Y_{cg} - Y_{arj}) - F_{ayj} (X_{cg} - X_{arj})$$

$$M''_{axj} = q_j \; S \; b \; (C_{l\beta j} \; \beta_j)$$

$$M''_{ayj} = q_j \; S \; \bar{c} \; (C_{moj} + C_{m\alpha j} \; \alpha_j)$$

$$M_{azj}'' = q_j \; S \; b \; C_{n\beta j}$$

$$M_{axj} = M_{axj}' + M_{axj}''$$

$$M_{ayj} = M_{ayj}' + M_{ayj}''$$

$$M_{azj} + M_{azj}' + M_{azj}''$$

Where:

$C_{l\delta a}$, $C_{n\delta a}$, $C_{l\beta j}$,

= aero coefficients for station j

δ_a = aileron deflection

4.7 Engine Moments

$$M_{txj}' = -T_{ybj} \; (\bar{Z}_{ej} - \bar{Z}_{cg}) + T_{zbj} \; (\bar{Y}_{ej} - \bar{Y}_{cg})$$

$$M_{tyj}' = -T_{zbj} \; (\bar{X}_{ej} - \bar{X}_{cg}) + T_{xbj} \; (\bar{Z}_{ej} - \bar{Z}_{cg})$$

$$M_{tzj}' = -T_{xbj} \; (\bar{Y}_{ej} - \bar{Y}_{cg}) + T_{ybj} \; (\bar{X}_{ej} - \bar{X}_{cg})$$

$$M_{txj}'' = -T_{ybj} \sum_{i=1}^{M3} \phi_{zij}\, q_i + T_{zbj} \sum_{i=1}^{M3} \phi_{yij}\, q_i$$

$$M_{tyj}'' = -T_{zbj} \sum_{i=1}^{M3} \phi_{xij}\, q_i + T_{xbj} \sum_{i=1}^{M3} \phi_{zij}\, q_i$$

$$M_{tzj}'' = -T_{xbj} \sum_{i=1}^{M3} \phi_{yij}\, q_i + T_{ybj} \sum_{i=1}^{M3} \phi_{xij}\, q_i$$

$$M_{exj} = [\, F_{ezj} - m_{ej}(X_{ej} - X_{ecg})\ddot{\theta}_{pj}' \,]\,(Y_{ej} - Y_{cg}) - [F_{eyj} + m_{ej}(X_{ej} - X_{ecgj})\ddot{\theta}_{yj}'](Z_{ej} - Z_{cg})$$

$$M_{eyj} = F_{exj}(Z_{ej} - Z_{ecgj}) - [F_{ezj} - m_{ej}(X_{ej} - X_{ecgj})\ddot{\theta}_{pj}'](X_{ej} - X_{cg})$$

$$M_{ezj} = [F_{eyj} + m_{ej}(X_{ej} - X_{ecgj})\ddot{\theta}_{yj}'](X_{ej} - X_{cg}) - F_{exj}(Y_{ej} - Y_{cg})$$

$$M_{txj} = M_{txj}' + M_{txj}'' + M_{exj}$$

$$M_{tyj} = M_{tyj}' + M_{tyj}'' + M_{eyj}$$

$$M_{tzj} = M_{tzj}' + M_{tzj}'' + M_{ezj}$$

Where:

I_{ej} = moment of inertia of engine j

4.8 Slosh Moments

$$M_{sxj}' = A_y m_{sj} \lambda_{zj} - A_z m_{sj} \lambda_{yj}$$

$$M_{syj}' = A_z m_{sj} \lambda_{xj} - A_x m_{sj} \lambda_{zi}$$

$$M_{szj}' = A_x m_{sj} \lambda_{yj} - A_y m_{sj} \lambda_{xi}$$

$$M_{sxj}'' = F_{syj} (\bar{Z}_{cg} - \bar{Z}_{sj}) - F_{szj} (\bar{Y}_{cg} - \bar{Y}_{sj})$$

$$M_{syj}'' = F_{szj} (\bar{X}_{cg} - \bar{X}_{sj}) - F_{sxj} (\bar{Z}_{cg} - Z_{sj})$$

$$M_{szj}'' = F_{sxj} (\bar{Y}_{cg} - \bar{Y}_{sj}) - F_{syj} (\bar{X}_{cg} - \bar{X}_{sj})$$

$$M_{sxj} = M_{sxj}' + M_{sxj}''$$

$$M_{syj} = M_{syj}' + M_{syj}''$$

$$M_{szj} = M_{szj}' + M_{szj}''$$